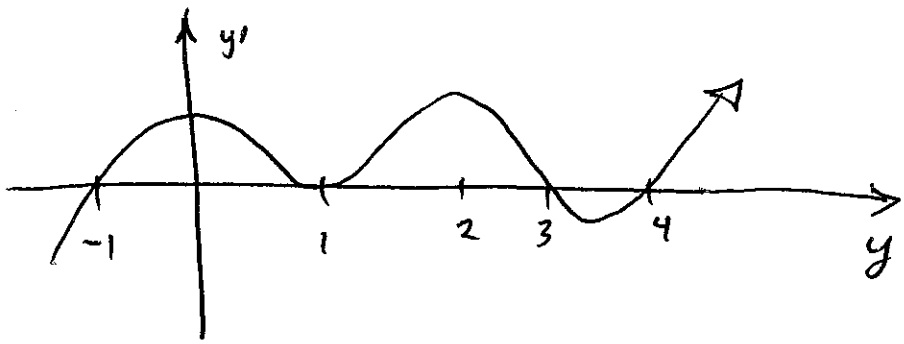
Extra Practice: Autonomous D.E.s

1. Given below is the graph of $y' = f(y)$ (that is, the "phase plane"). From this graph, answer the following:
   (a) Find the equilibrium solutions, if any. Label them as either **stable**, **unstable**, or **semistable**.
   (b) Find the interval(s) (in time) on which y is increasing and decreasing:
   (c) Find the interval(s) (in time) on which y is concave up or concave down.
   (d) Sketch the solutions in the (t, y) plane- be sure to keep in mind the previous answers.

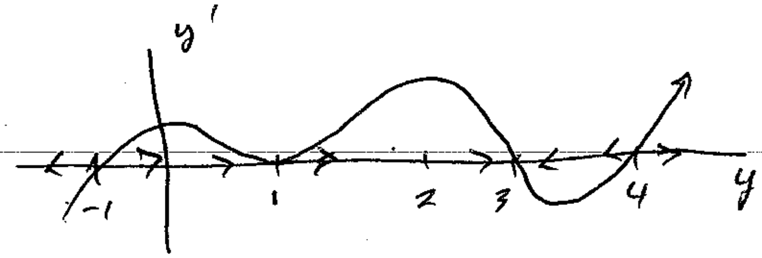


2. Draw a phase plane (y, y') if you want to build an autonomous D.E. with all the following properties, then write the model as $y' = f(y)$:
   - If the population is above 5 units, the population will decrease.
   - The population is at equilibrium at 3 units.
   - The population increases if it is below 3 units.
   - The population is at equilibrium at 0 units.
   - The population increases if it has negative units (whatever that means).

3. True or False, and explain:
   (a) Equilibrium solutions are constants.
   (b) If $y' = \sin(y)$, then the solution to the DE may be periodic.

Solns to extra Practice, Auton. DEs:

1.



unstable, semistable, stable, unstable

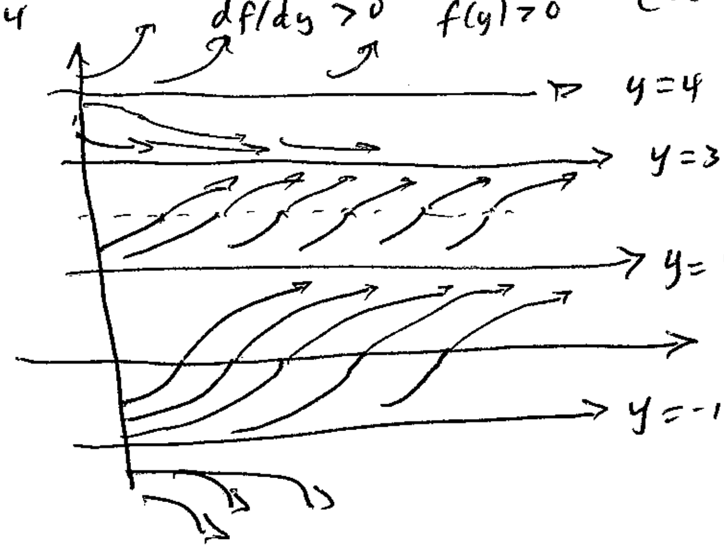
(a) The equilibria are the y-intercepts: $-1, 1, 3, 4$.

(b) y is inc (in t) when y' is pos: $-1 < y \leq 1$, $1 < y < 2$, $y > 4$

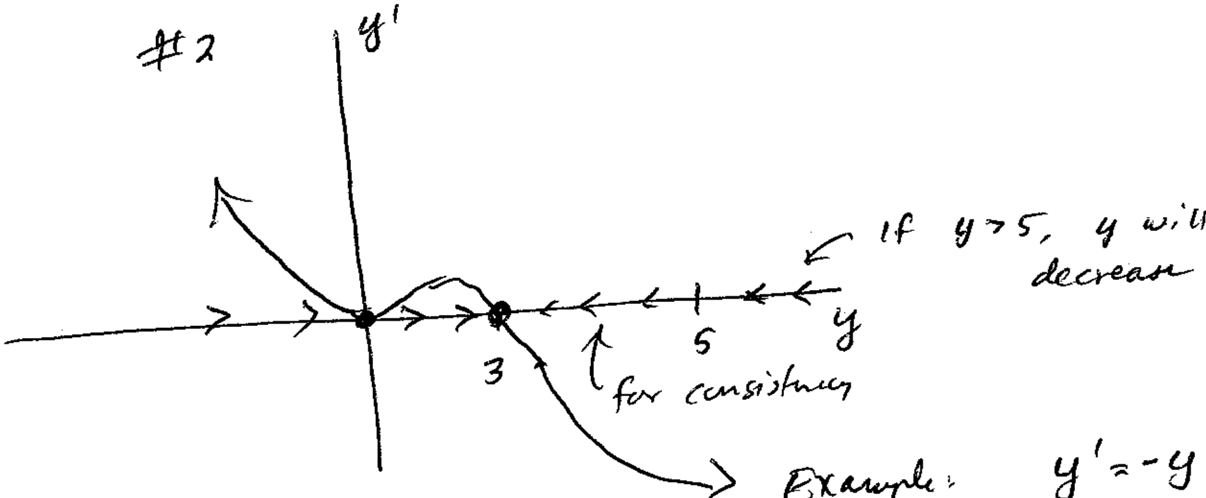
dec (" ") " " " neg: $y < -1$, $3 < y < 4$

(c) y is CU: $\frac{df}{dy} \cdot f(y) > 0$

$y < -1$:	$\frac{df}{dy} > 0$,	$f(y) < 0$,	<u>CD</u>
$-1 < y < 0$:	$\frac{df}{dy} > 0$,	$f(y) > 0$,	CU
$0 < y < 1$:	$\frac{df}{dy} < 0$,	$f(y) > 0$,	CD
$1 < y < 2$	$\frac{df}{dy} > 0$	$f(y) > 0$,	CU
$2 < y < 3$	$df/dy < 0$,	$f(y) > 0$,	CD
$3 < y < 3.5$	$df/dy < 0$	$f(y) < 0$	CU
$3.5 < y < 4$	$df/dy > 0$	$f(y) < 0$	CD
$y > 4$	$df/dy > 0$	$f(y) > 0$	CU



#2



<u>Example:</u> $y' = -y^2(y-3)$

#3. (a) True.

(b) False. $y(t)$ must be increasing, decreasing or constant - y cannot oscillate.