Practice with Integrals

Work out the following antiderivatives. There is a "hint sheet" attached. For the solutions, see the Maple file online.

1. $\displaystyle\int \frac{2x^2 - x + 4}{x^3 + 4x}\,dx$

2. $\displaystyle\int \mathrm{e}^{2\theta}\sin(3\theta)\,d\theta$

3. $\displaystyle\int \frac{1}{y(2-y)}\,dy$

4. $\displaystyle\int t^2\cos(3t)\,dt$

5. $\displaystyle\int x^3\mathrm{e}^{x^2}\,dx$

6. $\displaystyle\int x5^x\,dx$

7. $\displaystyle\int \frac{x-1}{x^2+1}\,dx$

8. $\displaystyle\int \frac{1}{x\sqrt{x+1}}\,dx$

9. $\displaystyle\int y\sinh(y)\,dy$

10. $\displaystyle\int \frac{dx}{x^4 - x^2}$

11. $\displaystyle\int \frac{t^2}{t+4}\,dt$

12. $\displaystyle\int \cos(\ln(x))\,dx$

13. $\displaystyle\int \frac{x-1}{x+4}\,dx$

14. $\displaystyle\int \tan^{-1}(x)\,dx$

Hint Sheet: Integration Practice

The Maple command for each integral is given so that you can check your answer.

1. $\displaystyle\int \frac{2x^2 - x + 4}{x^3 + 4x}\,dx$

   Use Partial Fraction Decomposition,

$$\frac{2x^2 - x + 4}{x^3 + 4x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

   Once you've found A, B, C, break the integral up:

$$A\int \frac{1}{x}\,dx + B\int \frac{x}{x^2+4}\,dx + C\int \frac{1}{x^2+4}\,dx$$

   Use u, du substitution for the second integral, and do some algebra on the last integral so you can use the inverse tangent as the antiderivative.

2. $\displaystyle\int \mathrm{e}^{2\theta}\sin(3\theta)\,d\theta$

   Integration by parts twice so that:

$$\int \mathrm{e}^{2\theta}\sin(3\theta)\,d\theta = \mathrm{e}^{2\theta}\,(...) - \frac{4}{9}\int \mathrm{e}^{2\theta}\sin(3\theta)\,d\theta$$

   Now solve for $\int \mathrm{e}^{2\theta}\sin(3\theta)\,d\theta$.

3. $\displaystyle\int \frac{1}{y(2-y)}\,dy$

   Use partial fractions:

$$\frac{1}{y(2-y)} = \frac{A}{y} + \frac{B}{2-y}$$

   then integrate. You might write it as $A/y - B/(y-2)$ to make it easier to integrate later.

4. $\displaystyle\int t^2\cos(3t)\,dt$

   Integration by parts; put t^2 in the middle column of the table so that the third derivative is zero.

5. $\displaystyle\int x^3\mathrm{e}^{x^2}\,dx$

   Do a u, du substitution first, with $u = x^2$, $du = 2x\,dx$:

$$\int x^3\mathrm{e}^{x^2}\,dx = \int x^2\mathrm{e}^{x^2}\,x\cdot dx = \frac{1}{2}\int u\mathrm{e}^u\,du$$

   Now do integration by parts on this.

6. $\int x5^x \, dx$

Do integration by parts, where the antiderivative of 5^x is $\frac{1}{\ln(5)}5^x$ (so put x in the middle column).

7. $\int \frac{x-1}{x^2+1} \, dx$

We can write this as:

$$\int \frac{x}{x^2+1} \, dx - \int \frac{1}{x^2+1} \, dx$$

Use u, du substitution for the first integral, the second integral is already in a nice form.

8. $\int \frac{1}{x\sqrt{x+1}} \, dx$

We will do a u, du substitution first:

$$u = \sqrt{x+1} \qquad du = \frac{1}{2} \cdot \frac{1}{\sqrt{x+1}} \, dx \qquad \text{and} \qquad u^2 - 1 = x$$

Making these substitutions:

$$\int \frac{1}{x\sqrt{x+1}} \, dx = 2 \int \frac{1}{u^2-1} \, du$$

Factor the denominator and use partial fractions to finish up.

9. $\int y \sinh(y) \, dy$

Use integration by parts, where the antiderivative of $\sinh(y)$ is $\cosh(y)$, and the antiderivative of $\cosh(y)$ is $\sinh(y)$ (so put y in the middle column).

10. $\int \frac{dx}{x^4 - x^2}$

Factor the denominator, and use integration by parts. The x^2 term can be thought of as a doubled up linear factor, so we would use:

$$\frac{A}{x} + \frac{B}{x^2}$$

or as a single quadratic factor, in which case we would use:

$$\frac{Ax+B}{x^2}$$

(These are equivalent). In any event,

$$\frac{1}{x^2(x^2-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x+1}$$

11. $\int \frac{t^2}{t+4}\,dt$

The degree of the numerator is greater than the degree of the denominator. Use long division to get:

$$\frac{t^2}{t+4} = t^2 - 4 + \frac{16}{t+4}$$

12. $\int \cos(\ln(x))\,dx$

Use a u, du substitution, where $u = \ln(x)$, or $\mathrm{e}^u = x$. Therefore, $\mathrm{e}^u\,du = dx$ and making our substitutions:

$$\int \cos(\ln(x))\,dx = \int \mathrm{e}^u \cos(u)\,du$$

We will have to integrate this by parts twice (like problem 2).

13. $\int \frac{x-1}{x+4}\,dx$

The degree of the numerator is larger than or equal to the degree of the denominator, so perform long division first:

$$\frac{x-1}{x+4} = 1 - \frac{5}{x+4}$$

14. $\int \tan^{-1}(x)\,dx$

Use integration by parts, with $u = \tan^{-1}(x)$ and $dv = 1\,dx$.