Chapter 3, Sect 6

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Background

Solve: $ay'' + by' + cy = g(t)$.

Recall that the Method of Undetermined Coefficients exploited the linear operator acting on certain classes of functions. If the ansatz was part of the homogeneous solution, multiply by t until it is not.

Today: Variation of Parameters

This method works on the more general form:

$$y'' + p(t)y' + q(t)y = g(t)$$

The method assumes that

$$y_p = u_1(t)y_1(t) + u_2(t)y_2(t)$$

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where y_1, y_2 form a fundamental set to the **homogeneous** problem.
Note: We do not know how to get y_1, y_2 except for special cases...

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NOTE: For exams/quizzes I will give you the system of equations

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From that, we see

$$u_1 = \int \frac{-y_2 g(t)}{W(y_1, y_2)}\, dt \qquad\qquad u_2 = \int \frac{y_1 g(t)}{W(y_1, y_2)}\, dt$$

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NOTE: Do you need to "add C" to your integrals?

$$u_1 = \int \frac{-y_2 g(t)}{W(y_1, y_2)}\, dt \qquad u_2 = \int \frac{y_1 g(t)}{W(y_1, y_2)}\, dt$$

Use Variation of Parameters to find the general solution:

$$4y'' - 4y' - 8y = 8\mathrm{e}^{-t}$$

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We need $W(y_1, y_2)$:

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$$u_1' = \frac{-\mathrm{e}^{2t} 2\mathrm{e}^{-t}}{3\mathrm{e}^{t}} = -\frac{2}{3} \qquad u_2' = \frac{\mathrm{e}^{-t} 2\mathrm{e}^{-t}}{3\mathrm{e}^{t}} = \frac{2}{3}\mathrm{e}^{-3t}$$

Therefore, $u_1(t) = -\frac{2}{3}t$ and $u_2 = -\frac{2}{9}\mathrm{e}^{-3t}$.

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Note: Using Method of Undetermined Coefficients, we would have guessed the same?

$$u_1 = \int \frac{-y_2 g(t)}{W(y_1, y_2)}\, dt \qquad u_2 = \int \frac{y_1 g(t)}{W(y_1, y_2)}\, dt$$

Use Variation of Parameters to find the general solution:

$$y'' - 2y' + y = \frac{\mathrm{e}^t}{1 + t^2}$$

SOLUTION: $y_1 = \mathrm{e}^t$, $y_2 = t\mathrm{e}^t$, $W = \mathrm{e}^{2t}$.

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$$u_1' = \frac{-t\mathrm{e}^t \frac{\mathrm{e}^t}{(1+t^2)}}{\mathrm{e}^{2t}} = \frac{-t}{1+t^2} \Rightarrow u_1 = -\frac{1}{2}\ln(1+t^2)$$

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$$u_2' = \frac{\mathrm{e}^t \frac{\mathrm{e}^t}{(1+t^2)}}{\mathrm{e}^{2t}} = \frac{1}{1+t^2} \Rightarrow u_2 = \tan^{-1}(t)$$

Therefore, the full solution is:

$$u_1 = \int \frac{-y_2 g(t)}{W(y_1, y_2)}\, dt \qquad u_2 = \int \frac{y_1 g(t)}{W(y_1, y_2)}\, dt$$

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$$y'' - 2y' + y = \frac{e^t}{1+t^2}$$

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Therefore, the full solution is:

$$y(t) = e^t\left(C_1 + C_2 t\right) - \frac{1}{2}e^t \ln(1+t^2) + te^t \tan^{-1}(t)$$

$$u_1 = \int \frac{-y_2 g(t)}{W(y_1, y_2)}\, dt \qquad u_2 = \int \frac{y_1 g(t)}{W(y_1, y_2)}\, dt$$

Use the Variation of Parameters to solve

$$t^2 y'' - t(t+2)y' + (t+2)y = 2t^3 \qquad y_1(t) = t \quad y_2(t) = t\mathrm{e}^t$$

SOLUTION:

$$u_1 = \int \frac{-y_2 g(t)}{W(y_1, y_2)}\, dt \qquad u_2 = \int \frac{y_1 g(t)}{W(y_1, y_2)}\, dt$$

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SOLUTION: $g(t) = 2t$ and $W(y_1, y_2) = t^2\mathrm{e}^t$, so

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SOLUTION: $g(t) = 2t$ and $W(y_1, y_2) = t^2\mathrm{e}^t$, so

$$u_1' = \frac{-2t^2\mathrm{e}^t}{t^2\mathrm{e}^t} = -2$$

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SOLUTION: $g(t) = 2t$ and $W(y_1, y_2) = t^2\mathrm{e}^t$, so

$$u_1' = \frac{-2t^2\mathrm{e}^t}{t^2\mathrm{e}^t} = -2 \quad \Rightarrow \quad u_1 = -2t$$

Similarly,

$$u_2' =$$

$$u_1 = \int \frac{-y_2 g(t)}{W(y_1, y_2)}\, dt \qquad u_2 = \int \frac{y_1 g(t)}{W(y_1, y_2)}\, dt$$

Use the Variation of Parameters to solve

$$t^2 y'' - t(t+2)y' + (t+2)y = 2t^3 \qquad y_1(t) = t \quad y_2(t) = t\mathrm{e}^t$$

SOLUTION: $g(t) = 2t$ and $W(y_1, y_2) = t^2\mathrm{e}^t$, so

$$u_1' = \frac{-2t^2\mathrm{e}^t}{t^2\mathrm{e}^t} = -2 \quad \Rightarrow \quad u_1 = -2t$$

Similarly,

$$u_2' = \frac{2t^2}{t^2\mathrm{e}^t} = 2\mathrm{e}^{-t}$$

$$u_1 = \int \frac{-y_2 g(t)}{W(y_1, y_2)}\, dt \qquad u_2 = \int \frac{y_1 g(t)}{W(y_1, y_2)}\, dt$$

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Similarly,

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so

$$y_p =$$

$$u_1 = \int \frac{-y_2 g(t)}{W(y_1, y_2)}\, dt \qquad u_2 = \int \frac{y_1 g(t)}{W(y_1, y_2)}\, dt$$

Use the Variation of Parameters to solve

$$t^2 y'' - t(t+2)y' + (t+2)y = 2t^3 \qquad y_1(t) = t \quad y_2(t) = te^t$$

SOLUTION: $g(t) = 2t$ and $W(y_1, y_2) = t^2 e^t$, so

$$u_1' = \frac{-2t^2 e^t}{t^2 e^t} = -2 \quad \Rightarrow \quad u_1 = -2t$$

Similarly,

$$u_2' = \frac{2t^2}{t^2 e^t} = 2e^{-t} \quad \Rightarrow \quad u_2 = -2e^{-t}$$

so

$$y_p = (-2t)(t) + (-2e^{-t})(te^t) = -2t^2 - 2t \quad \Rightarrow y_p(t) = -2t^2$$