Homework
Replaces Section 3.7, 3.8

1. Our text (p. 198) states that

$$\frac{\mu}{\omega_0} = \left(1 - \frac{\gamma^2}{4km}\right)^{1/2} \approx 1 - \frac{1}{2} \cdot \frac{\gamma^2}{4km}$$

   How was this approximation made? (Hint: Linearize $\sqrt{1-x}$)

2. Show that the period of motion of an undamped vibration of a mass hanging from a vertical spring is $2\pi\sqrt{L/g}$

3. Using $A = R\cos(\delta), B = R\sin(\delta)$ (and $R = \sqrt{A^2 + B^2}$), and the trig identity:

$$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

   our text states that we can convert:

$$u = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

   to $u = R\cos(\omega_0 t - \delta)$. Show how this is done with the following:

   (a) $u = 2\cos(3t) + \sin(3t)$
   (b) $u = -2\pi\cos(\pi t) - \pi\sin(\pi t)$
   (c) $u = 5\sin(t/2) - \cos(t/2)$

4. Show that in the case of beating,

$$y'' + \omega_0^2 y = a\ \cos(\omega t)$$

   the general solution is indeed of the form

$$y(t) = c_1\sin(\omega_0 t) + c_2\cos(\omega_0 t) + \frac{a}{\omega_0^2 - \omega^2}\cos(\omega t)$$

   and then, using the formula

$$\cos(a) - \cos(b) = 2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

   show that zero initial conditions leads to the solution:

$$y(t) = \frac{2a}{\omega_0^2 - \omega^2}\sin\left(\frac{(\omega_0 - \omega)t}{2}\right)\sin\left(\frac{(\omega_0 + \omega)t}{2}\right)$$

5. (Exercise 7, Section 3.7- Set up by hand, let Maple solve it.)

6. (Exercise 9, Section 3.7- Set up by hand, let Maple solve it.)

7. Find the general solution of the given differential equation:

$$y'' + 3y' + 2y = \cos(t)$$

8. What is the *transient solution*? What is the *steady state solution*? (See section 3.8)

9. Pictured below are the graphs of several solutions to the differential equation:

$$y'' + by' + cy = \cos(\omega t)$$

Match the figure to the choice of parameters:

Choice	b	c	ω
(A)	5	3	1
(B)	1	3	1
(C)	5	1	3
(D)	1	1	3

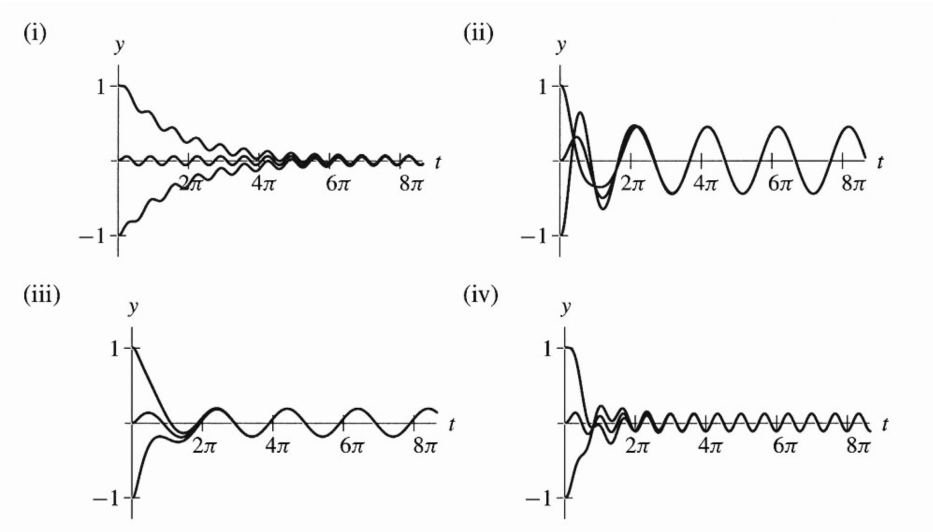

Figure 2: Figures for homework problem 2. Match each figure with the appropriate choice of constants.

10. Recall that

$$\text{Real}(e^{i\theta}) = \cos(\theta) \qquad \text{Imag}(e^{i\theta}) = \sin(\theta)$$

Show that, given the DE below we can use the ansatz $y_p = Ae^{3ti}$ (the real part),

$$y'' + 4y = 2\cos(3t)$$

and we will get the particular solution,

$$A = -\frac{2}{5} \qquad \Rightarrow \qquad y_p(t) = -\frac{2}{5}\cos(3t)$$

11. Fill in the question marks with the correct expression:

    Given the undamped second order differential equation, $y'' + \omega_0^2 y = A\cos(\omega t)$, we see "beating" if ?????????????. In particular, the longer period wave has a period that gets ?????????????? as $\omega \to \omega_0$, and its amplitude gets ?????????????

12. Find the solution to $y'' + 9y = 2\cos(3t)$, $y(0) = 0$, $y'(0) = 0$ by first solving the more general equation: $y'' + 9y = 2\cos(at)$, $y(0) = 0$, $y'(0) = 0$, then take the limit of your solution as $a \to 3$.

13. (Exercise 6, Section 3.8: Do this one by hand.)

14. (Exercise 10, Section 3.8: Verify your solution with Maple).

15. (Exercise 17, Section 3.8: Set up by hand, have Maple compute the amplitude in terms of ω and plot.)