

Maple Worksheet to show solutions to Exercises 23 and 24 in Math 244, Chapter 3.3:

First, define the differential equation. Note how we define the second derivative:

```
> DE23:=3*diff(u(t),t$2)-diff(u(t),t)+2*u(t)=0;
```

$$DE23 := 3 \left(\frac{d^2}{dt^2} u(t) \right) - \left(\frac{d}{dt} u(t) \right) + 2 u(t) = 0 \quad (1)$$

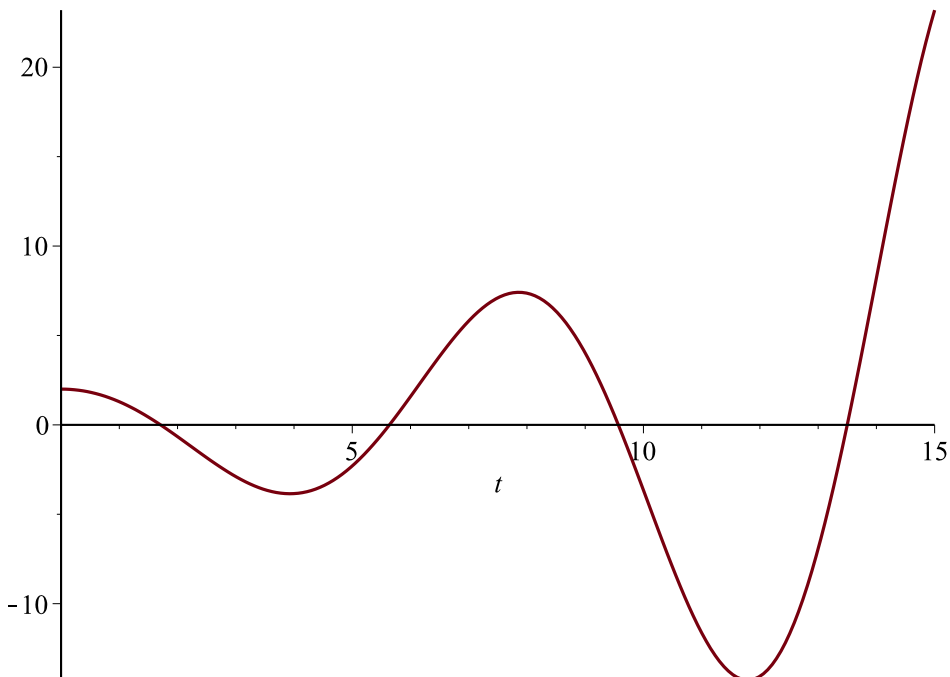
Next, we solve the DE using the "dsolve" command. Two things to note here: How we define the initial conditions (in particular, the initial velocity is defined using the D notation). Getting the "right hand side" of the equation, we put the solution into the variable U.

```
> U:=rhs( dsolve( { DE23, u(0)=2, D(u)(0)=0 },u(t)) );
```

$$U := -\frac{2}{23} \sqrt{23} e^{\frac{1}{6}t} \sin\left(\frac{1}{6} \sqrt{23} t\right) + 2 e^{\frac{1}{6}t} \cos\left(\frac{1}{6} \sqrt{23} t\right) \quad (2)$$

Plot the function. In particular, get an estimate for the solution (in this case, $|u(t)|=10$ somewhere between times 10 and 12).

```
> plot(U,t=0..15);
```



Now find the numerical solution using the "fsolve" command:

```
> fsolve(U=-10,t=10..12);
```

10.75977055

(3)

Now we'll solve Exercise 24 in much the same way:

```
> DE24:=5*diff(u(t),t$2)+2*diff(u(t),t)+7*u(t)=0;
```

$$DE24 := 5 \left(\frac{d^2}{dt^2} u(t) \right) + 2 \left(\frac{d}{dt} u(t) \right) + 7 u(t) = 0$$

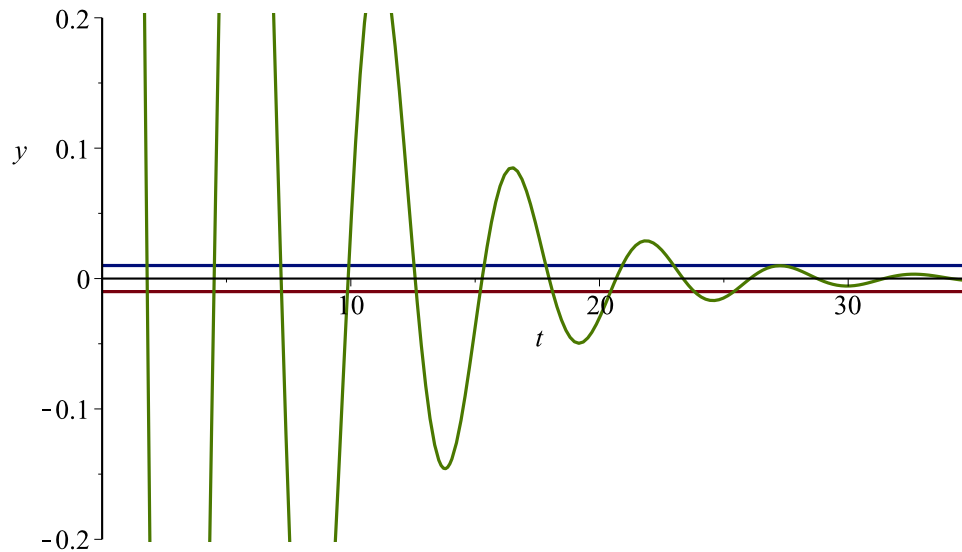
(4)

```
> U:=rhs( dsolve( { DE24, u(0)=2, D(u)(0)=1 },u(t)) );
```

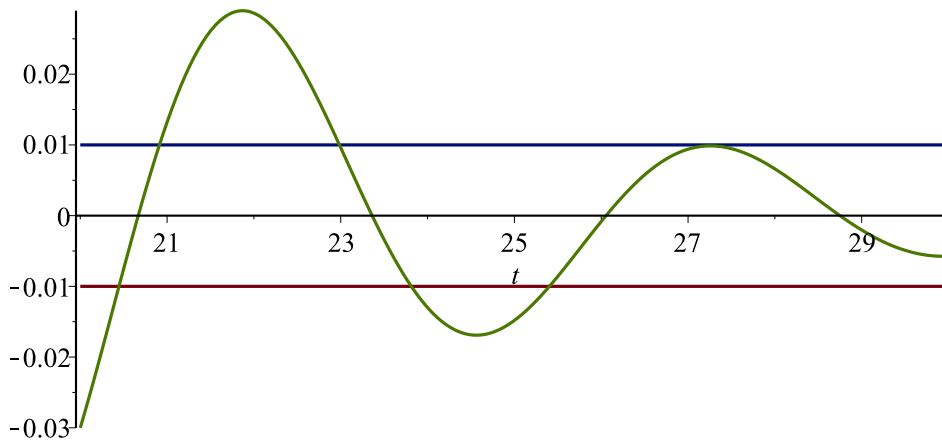
$$U := \frac{7}{34} \sqrt{34} e^{-\frac{1}{5}t} \sin\left(\frac{1}{5} \sqrt{34} t\right) + 2 e^{-\frac{1}{5}t} \cos\left(\frac{1}{5} \sqrt{34} t\right)$$

(5)

```
> plot({U,0.01,-0.01},t=0..35,y=-0.2..0.2);
```



```
> plot({U,0.01,-0.01},t=20..30);
```



```
> fsolve(U=-0.01,t=25..27);  
25.40299042
```

(6)