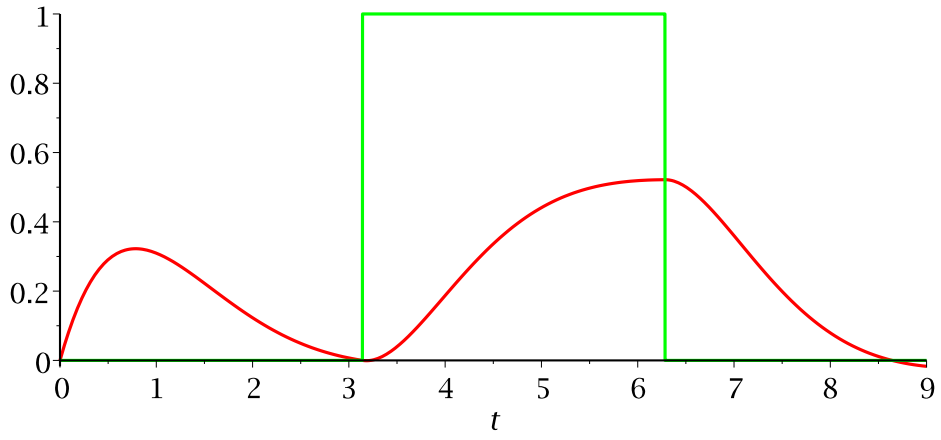
```
> with(plots):
> Eqn02:=diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=Heaviside(t-Pi)-
  Heaviside(t-2*Pi);
> Y1:=dsolve({Eqn02,y(0)=0,D(y)(0)=1},y(t));
> A:=plot(rhs(Y1),t=0..9,color=red):
> B:=plot(Heaviside(t-Pi)-Heaviside(t-2*Pi),t=0..9, color=green):
> display(A,B);
```

$$Eqn02 := \frac{d^2}{dt^2} y(t) + 2\left(\frac{d}{dt} y(t)\right) + 2\,y(t) = \text{Heaviside}(t-\pi) - \text{Heaviside}(t-2\pi)$$

$$Y1 := y(t) = e^{-t}\sin(t) + \frac{1}{2}\,\text{Heaviside}(t-2\pi)\,(\cos(t)+\sin(t))\,e^{-t+2\pi}$$

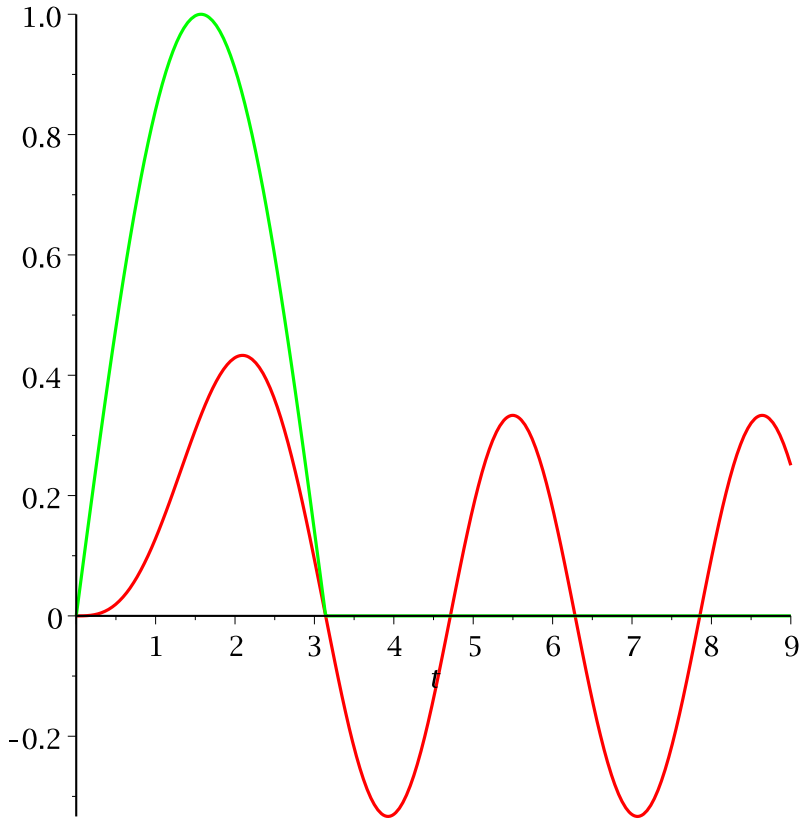
$$-\frac{1}{2}\,\text{Heaviside}(t-2\pi) + \frac{1}{2}\left(1+(\cos(t)+\sin(t))\,e^{-t+\pi}\right)\text{Heaviside}(t-\pi)$$




```
> Eqn04:=diff(y(t),t$2)+4*y(t)=sin(t)+Heaviside(t-Pi)*sin(t-Pi);
> Y4:=dsolve({Eqn04,y(0)=0,D(y)(0)=0},y(t));
> A:=plot(rhs(Y4),t=0..9,color=red):
> B:=plot(sin(t)+Heaviside(t-Pi)*sin(t-Pi),t=0..9, color=green):
> display(A,B);
```

$$Eqn04 := \frac{d^2}{dt^2} y(t) + 4\,y(t) = \sin(t) - \sin(t)\,\text{Heaviside}(t-\pi)$$

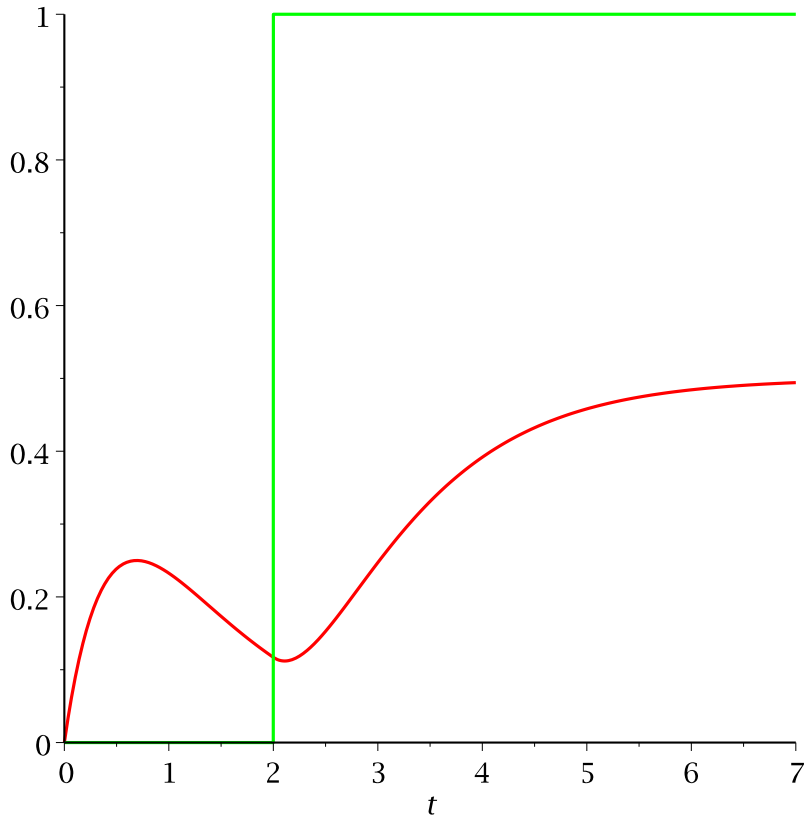
$$Y4 := y(t) = -\frac{1}{6}\sin(2t) + \frac{1}{6}\,(-2\sin(t) - \sin(2t))\,\text{Heaviside}(t-\pi) + \frac{1}{3}\sin(t)$$




```
> Eqn06:=diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=Heaviside(t-2);
> Y6:=dsolve({Eqn06,y(0)=0,D(y)(0)=1},y(t));
> A:=plot(rhs(Y6),t=0..7,color=red):
> B:=plot(Heaviside(t-2),t=0..7, color=green):
> display(A,B);
```

$$Eqn06 := \frac{d^2}{dt^2} y(t) + 3\left(\frac{d}{dt} y(t)\right) + 2\,y(t) = \text{Heaviside}(t-2)$$

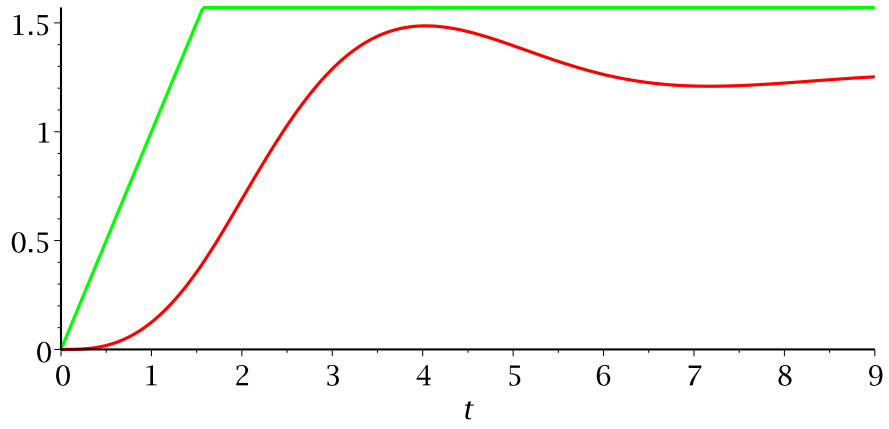
$$Y6 := y(t) = \frac{1}{2}\,\text{Heaviside}(t-2) - \text{Heaviside}(t-2)\,e^{-t+2} + \frac{1}{2}\,\text{Heaviside}(t-2)\,e^{4-2t} - e^{-2t} + e^{-t}$$




```
> Eqn08:=diff(y(t),t$2)+diff(y(t),t)+(5/4)*y(t)=t-Heaviside(t-Pi/2)
  *(t-Pi/2);
> Y8:=dsolve({Eqn08,y(0)=0,D(y)(0)=0},y(t));
> A:=plot(rhs(Y8),t=0..9,color=red):
> B:=plot(t-Heaviside(t-Pi/2)*(t-Pi/2),t=0..9, color=green):
> display(A,B);
```

$$Eqn08 := \frac{d^2}{dt^2} y(t) + \frac{d}{dt} y(t) + \frac{5}{4} y(t) = t - \text{Heaviside}\left(t - \frac{1}{2}\pi\right)\left(t - \frac{1}{2}\pi\right)$$

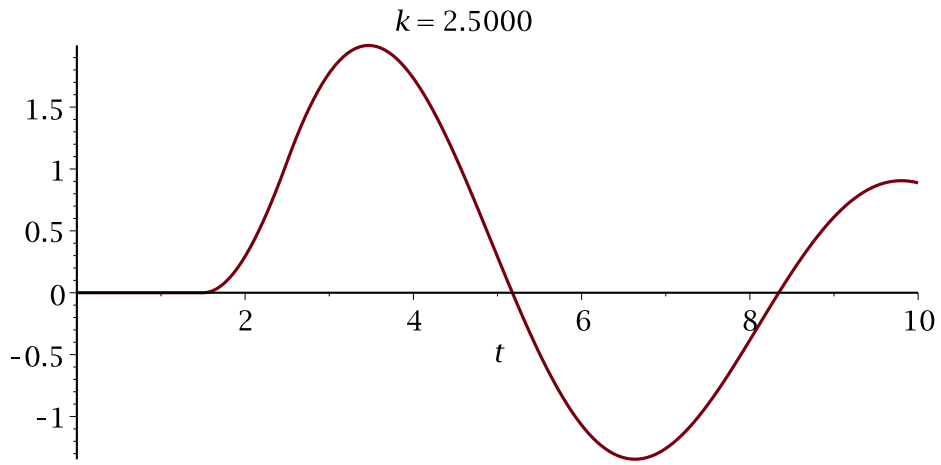
$$Y8 := y(t) = -\frac{12}{25} e^{-\frac{1}{2}t} \sin(t) + \frac{16}{25} e^{-\frac{1}{2}t} \cos(t) - \frac{12}{25}\left(\cos(t) + \frac{4}{3}\sin(t)\right) \text{Heaviside}\left(t - \frac{1}{2}\pi\right) e^{-\frac{1}{2}t + \frac{1}{4}\pi} + \frac{1}{25}(-20t + 10\pi + 16)\,\text{Heaviside}\left(t - \frac{1}{2}\pi\right) + \frac{4}{5}t - \frac{16}{25}$$




```
> Eqn16:=diff(y(t),t$2)+(1/4)*diff(y(t),t)+y(t)=k*(Heaviside(t-3/2)
  -Heaviside(t-5/2));
> Y16:=dsolve({Eqn16,y(0)=0,D(y)(0)=0},y(t));
  animate(plot,[rhs(Y16),t=0..10],k=0.1..2.5);
```

$$Eqn16 := \frac{d^2}{dt^2} y(t) + \frac{1}{4} \frac{d}{dt} y(t) + y(t) = k \left(\text{Heaviside}\left(t - \frac{3}{2} \right) - \text{Heaviside}\left(t - \frac{5}{2} \right) \right)$$

$$Y16 := y(t) = \left(\frac{1}{21} \sqrt{7}\, \text{Heaviside}\left(t - \frac{5}{2} \right) e^{-\frac{1}{8} t + \frac{5}{16}} \sin\left(\frac{1}{16} (6t - 15) \sqrt{7} \right) - \frac{1}{21} \sqrt{7}\, \text{Heaviside}\left(t - \frac{3}{2} \right) e^{-\frac{1}{8} t + \frac{3}{16}} \sin\left(\frac{1}{16} (6t - 9) \sqrt{7} \right) + \text{Heaviside}\left(t - \frac{5}{2} \right) e^{-\frac{1}{8} t + \frac{5}{16}} \cos\left(\frac{1}{16} (6t - 15) \sqrt{7} \right) - \text{Heaviside}\left(t - \frac{3}{2} \right) e^{-\frac{1}{8} t + \frac{3}{16}} \cos\left(\frac{1}{16} (6t - 9) \sqrt{7} \right) - \text{Heaviside}\left(t - \frac{5}{2} \right) + \text{Heaviside}\left(t - \frac{3}{2} \right) \right) k$$

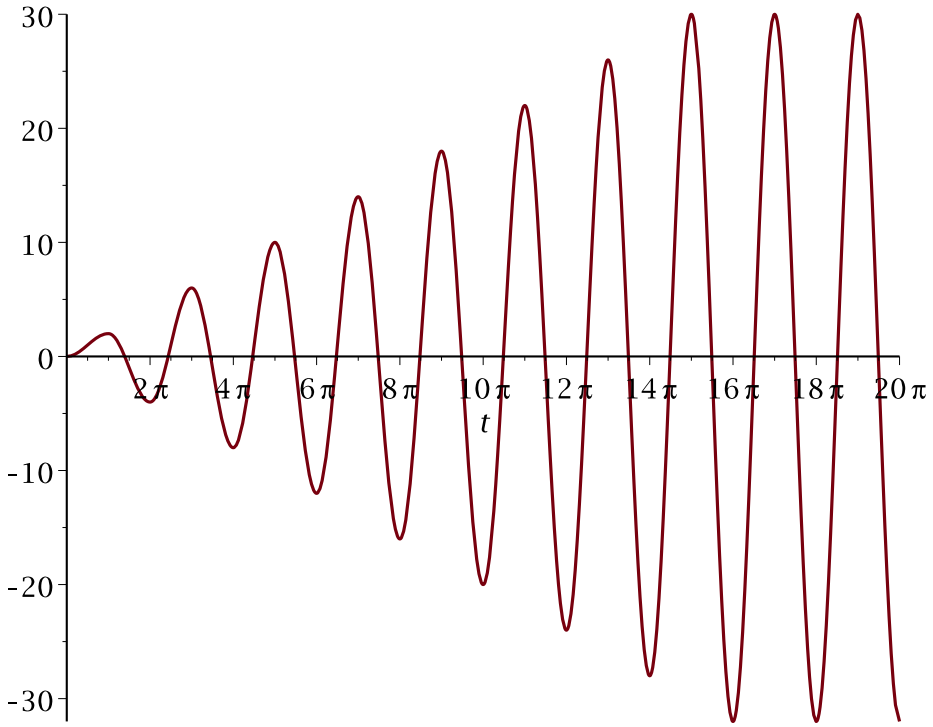



```
> N:=15;
> Eqn19:=diff(y(t),t$2)+y(t)=Heaviside(t)+2*sum((-1)^k*Heaviside(t-
  k*Pi),k=1..N);
> Y1:=dsolve({Eqn19,y(0)=0,D(y)(0)=0},y(t),method=laplace);
> plot(rhs(Y1),t=0..20*Pi);
> N:=30;
> Eqn19:=diff(y(t),t$2)+y(t)=Heaviside(t)+2*sum((-1)^k*Heaviside(t-
  k*Pi),k=1..N);
> Y1:=dsolve({Eqn19,y(0)=0,D(y)(0)=0},y(t),method=laplace);
> plot(rhs(Y1),t=0..50*Pi);
```

$$N := 15$$

$$Eqn19 := \frac{d^2}{dt^2} y(t) + y(t) = \text{Heaviside}(t) - 2\,\text{Heaviside}(t-\pi) + 2\,\text{Heaviside}(t - 2\pi) - 2\,\text{Heaviside}(-3\pi + t) + 2\,\text{Heaviside}(-4\pi + t) - 2\,\text{Heaviside}(-5\pi + t) + 2\,\text{Heaviside}(-6\pi + t) - 2\,\text{Heaviside}(-7\pi + t) + 2\,\text{Heaviside}(-8\pi + t) - 2\,\text{Heaviside}(-9\pi + t) + 2\,\text{Heaviside}(-10\pi + t) - 2\,\text{Heaviside}(-11\pi + t) + 2\,\text{Heaviside}(-12\pi + t) - 2\,\text{Heaviside}(-13\pi + t) + 2\,\text{Heaviside}(-14\pi + t) - 2\,\text{Heaviside}(-15\pi + t)$$

$$Y1 := y(t) = 1 - \cos(t) - 4\cos\left(\frac{1}{2}t\right)^2 (\text{Heaviside}(-15\pi + t) + \text{Heaviside}(t-\pi) + \text{Heaviside}(-3\pi + t) + \text{Heaviside}(-5\pi + t) + \text{Heaviside}(-7\pi + t) + \text{Heaviside}(-9\pi + t) + \text{Heaviside}(-11\pi + t) + \text{Heaviside}(-13\pi + t)) + 4\sin\left(\frac{1}{2}t\right)^2 (\text{Heaviside}(-14\pi + t) + \text{Heaviside}(t - 2\pi) + \text{Heaviside}(-4\pi + t) + \text{Heaviside}(-6\pi + t) + \text{Heaviside}(-8\pi + t) + \text{Heaviside}(-10\pi + t) + \text{Heaviside}(-12\pi + t))$$

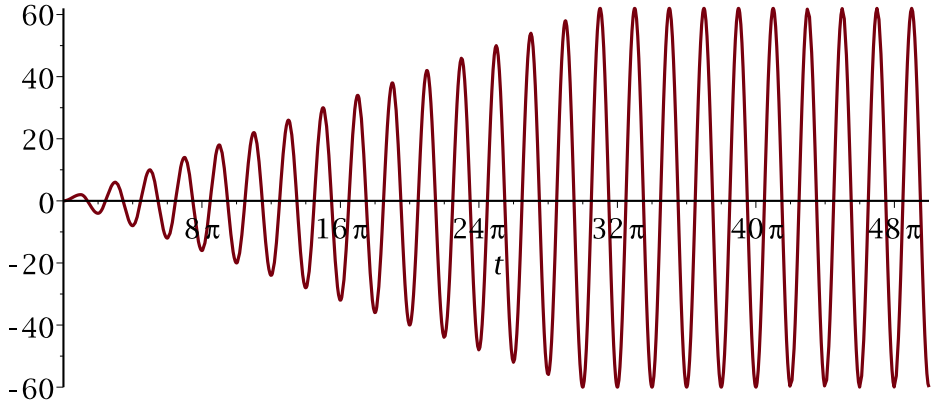


$$N := 30$$

$$Eqn19 := \frac{d^2}{dt^2} y(t) + y(t) = 2\,\text{Heaviside}(-30\pi + t) - 2\,\text{Heaviside}(-29\pi + t) + 2\,\text{Heaviside}(-28\pi + t) - 2\,\text{Heaviside}(-27\pi + t) + 2\,\text{Heaviside}(-26\pi + t) - 2\,\text{Heaviside}(-25\pi + t) - 2\,\text{Heaviside}(-15\pi + t) + 2\,\text{Heaviside}(-14\pi + t) - 2\,\text{Heaviside}(-13\pi + t) + 2\,\text{Heaviside}(-12\pi + t) - 2\,\text{Heaviside}(-11\pi + t) + 2\,\text{Heaviside}(-10\pi + t) - 2\,\text{Heaviside}(-9\pi + t) + 2\,\text{Heaviside}(-8\pi + t) - 2\,\text{Heaviside}(-7\pi + t) + 2\,\text{Heaviside}(-6\pi + t) - 2\,\text{Heaviside}(-5\pi + t) + 2\,\text{Heaviside}(-4\pi + t) + 2\,\text{Heaviside}(-16\pi + t) - 2\,\text{Heaviside}(-17\pi + t) + 2\,\text{Heaviside}(-18\pi + t) - 2\,\text{Heaviside}(-3\pi + t) + \text{Heaviside}(t) - 2\,\text{Heaviside}(-19\pi + t) + 2\,\text{Heaviside}(-20\pi + t) - 2\,\text{Heaviside}(-21\pi + t) + 2\,\text{Heaviside}(-22\pi + t) - 2\,\text{Heaviside}(-23\pi + t) + 2\,\text{Heaviside}(-24\pi + t) - 2\,\text{Heaviside}(t - \pi) + 2\,\text{Heaviside}(t - 2\pi)$$

$$Y1 := y(t) = 1 - \cos(t) - 4\cos\left(\frac{1}{2}\,t\right)^2 (\text{Heaviside}(-29\pi + t) + \text{Heaviside}(-15\pi + t) + \text{Heaviside}(-13\pi + t) + \text{Heaviside}(-11\pi + t) + \text{Heaviside}(-9\pi + t) + \text{Heaviside}(-7\pi + t) + \text{Heaviside}(-5\pi + t) + \text{Heaviside}(-3\pi + t)$$

$$+\text{Heaviside}(t-\pi)+\text{Heaviside}(-21\pi+t)+\text{Heaviside}(-17\pi+t)$$
$$+\text{Heaviside}(-23\pi+t)+\text{Heaviside}(-25\pi+t)+\text{Heaviside}(-19\pi+t)$$
$$+\text{Heaviside}(-27\pi+t))+4\sin\left(\frac{1}{2}t\right)^2(\text{Heaviside}(-30\pi+t)+\text{Heaviside}(-14\pi+t)+\text{Heaviside}(-12\pi+t)+\text{Heaviside}(-10\pi+t)+\text{Heaviside}(-8\pi+t)+\text{Heaviside}(-6\pi+t)+\text{Heaviside}(-4\pi+t)+\text{Heaviside}(t-2\pi)$$
$$+\text{Heaviside}(-24\pi+t)+\text{Heaviside}(-20\pi+t)+\text{Heaviside}(-28\pi+t)$$
$$+\text{Heaviside}(-18\pi+t)+\text{Heaviside}(-22\pi+t)+\text{Heaviside}(-26\pi+t)$$
$$+\text{Heaviside}(-16\pi+t))$$




```
> with(inttrans):
> N:=5;
> Eqn20:=diff(y(t),t$2)+(1/10)*diff(y(t),t)+y(t)=Heaviside(t)+
> 2*Sum((-1)^k*Heaviside(t-k*Pi),k=1..N);
> Y1:=laplace(Eqn20,t,s);
> Y2:=subs(y(0)=0,D(y)(0)=0,Y1);
> Y3:=solve(Y2,laplace(y(t),t,s));
> Y4:=invlaplace(Y3,s,t);
> plot(Y4,t=0..100);
```

$$N := 5$$

$$Eqn20 := \frac{d^2}{dt^2}y(t)+\frac{1}{10}\frac{d}{dt}y(t)+y(t)=\text{Heaviside}(t)+2\left(\sum_{k=1}^{5}(-1)^k\text{Heaviside}(-\pi k+t)\right)$$

$$Y1 := s^2\,laplace(y(t),t,s)-D(y)(0)-s\,y(0)+\frac{1}{10}s\,laplace(y(t),t,s)-\frac{1}{10}y(0)$$

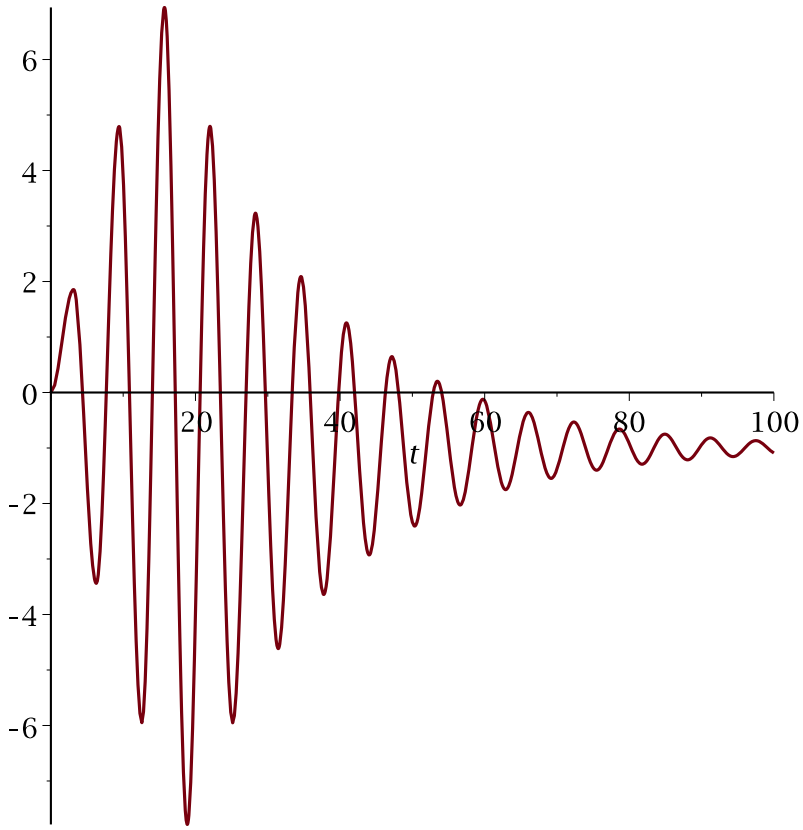
$$+ laplace(y(t),\, t,\, s) = \frac{1 + 2\left(\sum_{k=1}^{5} (-1)^k e^{-s\pi k}\right)}{s}$$

$$Y2 := s^2\, laplace(y(t),\, t,\, s) + \frac{1}{10}\, s\, laplace(y(t),\, t,\, s) + laplace(y(t),\, t,\, s)$$

$$= \frac{1 + 2\left(\sum_{k=1}^{5} (-1)^k e^{-s\pi k}\right)}{s}$$

$$Y3 := \frac{10\left(1 + 2\left(\sum_{k=1}^{5} (-1)^k e^{-s\pi k}\right)\right)}{s\,(10\,s^2 + s + 10)}$$

$$Y4 := 1 + \frac{2}{399} \sum_{k=1}^{5} \text{Heaviside}(-\pi k + t)\, (-1)^k \left(399 - e^{\frac{1}{20} k\pi - \frac{1}{20} t} \sqrt{399}\left(\sin\left(\frac{1}{20}\sqrt{399}\,(-\pi k + t)\right) + \sqrt{399}\cos\left(\frac{1}{20}\sqrt{399}\,(-\pi k + t)\right)\right)\right) - \frac{1}{399}\, e^{-\frac{1}{20} t}\left(\sqrt{399}\sin\left(\frac{1}{20}\sqrt{399}\, t\right) + 399\cos\left(\frac{1}{20}\sqrt{399}\, t\right)\right)$$


6
4
2
0
-2
-4
-6
20
40
60
80
100
t