Solutions: Conversions

1. Convert the following n^{th} order DEs to systems of first order.

   (a) $y'' + 3y' + 2y = 0$
   SOLUTION: Let $x_1 = y, x_2 = y'$. Then:

   $$\begin{aligned} x_1' &= x_2 \\ x_2' &= -2x_1 - 3x_2 \end{aligned}$$

   (b) $y''' = 2y - 3y'$
   SOLUTION: Let $x_1 = y, x_2 = y', x_3 = y''$. Then:

   $$\begin{aligned} x_1' &= x_2 \\ x_2' &= x_3 \\ x_3' &= 2x_1 - 3x_2 \end{aligned}$$

   (c) $2y'' + 6y' + y = 0$
   SOLUTION: Let $x_1 = y, x_2 = y'$. Then:

   $$\begin{aligned} x_1' &= x_2 \\ x_2' &= -(1/2)x_1 - 3x_2 \end{aligned}$$

   (d) $y'' + 5y = 0$
   SOLUTION: Let $x_1 = y, x_2 = y'$. Then:

   $$\begin{aligned} x_1' &= x_2 \\ x_2' &= -5x_1 \end{aligned}$$

   (e) $y^{(v)} = y - 3y' + ty'' + y''' - 3y^{(iv)}$
   SOLUTION: Let $x_1 = y, x_2 = y', x_3 = y'', x_4 = y^{'''}, x_5 = y^{(iv)}$. Then:

   $$\begin{aligned} x_1' &= x_2 \\ x_2' &= x_3 \\ x_3' &= x_4 \\ x_4' &= x_5 \\ x_5' &= x_1 - 3x_2 + tx_3 + x_4 - 3x_5 \end{aligned}$$

2. Convert the following systems of first order to an equivalent second order DE, if possible.

   (a) $\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 2x_1 + x_2 \end{aligned}$ Using Eqn 1, get $x_2 = \frac{1}{2}x_1' - \frac{1}{2}x_1$:

   $$\frac{1}{2}x_1'' - \frac{1}{2}x_1' = 2x_1 + \frac{1}{2}x_1' - \frac{1}{2}x_1 \quad \Rightarrow \quad x_1'' - 2x_1' - 3x_1 = 0$$

(b) $\begin{aligned} x_1' &= -2x_1 + x_2 \\ x_2' &= x_1 + x_2 \end{aligned}$ Using Eqn 1, get $x_2 = x_1' + 2x_1$:

$$x_1'' + 2x_1' = x_1 + x_1' + 2x_1 \quad \Rightarrow \quad x_1'' + x_1' - 3x_1 = 0$$

(c) $\begin{aligned} x_1' &= x_1 \\ x_2' &= 3x_2 \end{aligned}$

Since the DE for x_1 depends only on x_1 and the DE for x_2 depends only on x_2, we say that the system is **decoupled**. In that case, we cannot express the system as a single second order DE.