Basic Modeling Examples

Write down the differential equation for each situation, and give the solution (unless otherwise noted). The solutions are on the class website as a Maple document!

1. Suppose we borrow \$9000 for a car at an annual interest rate of 4%. We can afford payments of \$150 per month. Write the IVP for the amount we owe at year t, then solve it. Give the equation we would need to solve in order to ascertain when we would be paid in full (don't actually solve).

2. Suppose that we have fish in a pond (with no predators and plenty of food), and the initial population is 50 fish (to make the numbers work out nicely). (i) If the doubling time in the population is two years, give and solve the model using the exponential growth model. (ii) If fishing starts to harvest the fish at k fish per year (assume continuous harvesting, whatever that means), how does that change our model?

3. A cannonball is fired into the sky! It has a mass of 10 kg and with an initial speed of 10 m/s (recall that speed is the absolute value of velocity). Suppose air resistance is proportional to the velocity with a constant of 5. Determine the velocity of the cannonball (give the IVP and solve it). You might also recall that $g = 9.8$ m/s and our model gave "down" as positive.

4. Suppose a tank containing 100 gallons of pure water has a salty brine being pumped in at a rate of 3 gal/min, with incoming salt at 3 lbs per gallon. (i) If the well-mixed solution is being pumped out at a rate of 3 gal/min, find the IVP for the amount of salt in the tank at time t, and solve it. (ii) If the well-mixed solution is being pumped out at a rate of 4 gal/min, do the same thing (write the IVP and solve it).

5. Recall that Newton's Law of Cooling states that "the rate of change of the temperature of a body is proportional to the difference between the temperatures of the body and its environment". Write the DE for Newton's Law of Cooling and give the general solution.