Homework for 3.7

1. For practice with trig, 3.7.1-3.7.4, also given below. In each case, write the sum as $R\cos(\omega t - \delta)$

   3.7.1 $3\cos(2t) + 4\sin(2t)$

   3.7.2 $-\cos(t) + \sqrt{3}\sin(t)$

   3.7.3 $4\cos(3t) - 2\sin(3t)$

   3.7.4 $-2\cos(\pi t) - 3\sin(\pi t)$

2. Practice with the Model (metric system):

   (a) A spring with a 3-kg mass is held stretched 0.6 meters beyond its natural length by a force of 20 N. If the spring begins at its equilibrium position but a push gives it an initial velocity of 1.2 m/s, find the position of the mass after t seconds (assume no damping).

   (b) A spring with a 4-kg mass has a natural length of 1 meter, and is maintained stretched to a length of 1.3 m by a force of 24.3 N. If the spring is compressed to a length of 0.8 m and then released with zero velocity, find the position of the mass after t seconds (assume no damping).

   (c) A spring with a mass of 2 kg has damping constant 14 and a force of 6 N is required to keep the spring stretched 0.5 m beyond its natural length. The spring is then stretched to 1 m beyond its natural length and released. Find the position of the mass at any time t.

   (d) A spring with a mass of 3 kg has a damping constant 30 and spring constant 123. Find the position of mass at time t if it starts at equilibrium with a velocity of 2 m/s.

   (e) For the spring model above with a mass of 4 kg, find the damping constant that would produce critical damping.

   (f) A mass of 20 grams stretches a spring 5 cm. Suppose tha the mass is attached to a viscous damper with a constant damping constant of 400 dyn-s/cm (note: a dyne is a unit of force using centimeters-grams- seconds for units). If the mass is pulled down an additional 2 cm then released, find the IVP that governs the motion of the mass. (Calculator needed- g should be taken as 980).

   (g) Suppose we consider a mass-spring system with no damping (the damping constant is then 0), so that the differential equation expressing the motion of the mass can be modeled as

   $$mu'' + ku = 0$$

   Find value(s) of β so that $A\cos(\beta t)$ and $B\sin(\beta t)$ are each solutions to the homogeneous equation (for arbitrary values of A, B).

3. Practice with the model (in pounds, from the text)

   (a) A mass weighing 4 lb stretches a spring 2 in. Suppose that the mass is displaced an additional 6 in the positive direction and released. The mass is in a medium that exerts a viscous resistance of 6 lb when the mass has a velocity of 3 ft/s. Formulate the IVP that governs the motion of the mass. (Hint: All units should be consistent. When working with US units, use pounds, feet and seconds.)

   (b) A mass weighing 2 lb stretches a spring 6 in. If the mass is pulled down an additional 3 in and then released, and if there is no damping, determine the IVP that governs the motion of the mass. (You might re-read the hint for (1))

   (c) A mass of 20 grams stretches a spring 5 cm. Suppose tha the mass is attached to a viscous damper with a constant damping constant of 400 dyn-s/cm (note: a dyne is a unit of force using centimeters-grams- seconds for units). If the mass is pulled down an additional 2 cm then released, find the IVP that governs the motion of the mass.

   (d) A mass weighing 8 lbs stretches a spring $\frac{3}{2}$ in. The mass is attached to a damper with coefficient γ. Find γ so that the spring is *underdamped*, *critically damped*, *overdamped*.

Don't forget the Complex Exponentials handout!