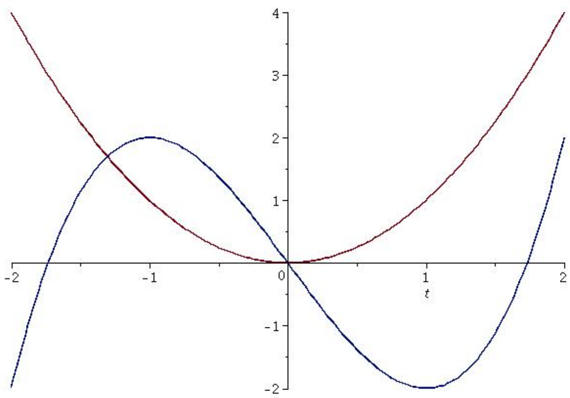
Math 244: Homework To Replace 7.1

1. If $x(t)$ and $y(t)$ are plotted below (versus time), plot the graph in the xy-plane. (Hint: You might think about some specific points in the $xy-$plane). Remember that at $t = -2$, you're starting all the way to the left, and at $t = 2$, you're all the way to the right (in the $t-$plane). You may assume that the $x-$coordinate is the parabola.



2. Exercise 22 (Section 7.1, p 363, tank mixing)

3. Solve the system of equations given by first converting it into a second order linear ODE (then use Chapter 3 methods):

   (a) $\begin{aligned} x' &= -2x + y \\ y' &= x - 2y \end{aligned}$ (b) $\begin{aligned} x' &= 2y \\ y' &= -2x \end{aligned}$

4. Give the solution to each system of equations. If it has an infinite number of solutions, give your answer in vector form:

$$\begin{aligned} 3x + 2y &= 1 \\ 2x - y &= 3 \end{aligned} \qquad \begin{aligned} 3x + 2y &= 1 \\ 6x + 4y &= 3 \end{aligned} \qquad \begin{aligned} 3x + 2y &= 1 \\ 6x + 4y &= 2 \end{aligned}$$

5. Let A, B be the matrices below. Compute the matrix operation listed.

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

   (a) $2A + B^T$ (b) BA (c) A^{-1}

6. Vectors and matrices might have complex numbers. If $z = 3 + 2i$ and vector $\mathbf{v} = [1 + i, 2 - 2i]^T$, then find the real part and the imaginary part of $z\mathbf{v}$.

7. Adding two vectors: Geometrically (and numerically) compute the following, where $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Be sure to draw each vector out, and see if you can see a pattern.

(a) $\mathbf{u}+\mathbf{v}$ (b) $\mathbf{u}-2\mathbf{v}$ (c) $\mathbf{u}+\frac{1}{2}\mathbf{v}$ (d) $-\mathbf{u}+\mathbf{v}$

8. Verify that $\mathbf{x}_1(t)$ below satisfies the DE: $\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \mathbf{x}$, $\quad \mathbf{x}_1(t) = e^{3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

9. Consider

$$\begin{aligned} x' &= 2x + 3y + 1 \\ y' &= x - y - 2 \end{aligned} .$$

First find the equilibrium solution, x_e, y_e. Then show that, if $u = x - x_e$ and $v = y - y_e$, then

$$\begin{aligned} u' &= 2u + 3v \\ v' &= u - v \end{aligned}$$

10. The four graphs in the figure show the direction fields for the four systems of differential equations below. Try to reason out which direction field goes with which system.

(a) $\begin{aligned} x' &= 3x - y \\ y' &= 4x - 2y \end{aligned}$ (c) $\begin{aligned} x' &= 2y \\ y' &= -2x \end{aligned}$

(b) $\begin{aligned} x' &= -2x + y \\ y' &= x - 2y \end{aligned}$ (d) $\begin{aligned} x' &= -x - 4y \\ y' &= x - y \end{aligned}$