L005 Section 6.2 Solving DEs with Laplace Transforms

- From the previous video,

$$\mathcal{L}(y'(t)) = sY(s) - y(0)$$

$$\mathcal{L}(y''(t)) = s^2Y(s) - sy(0) - y'(0)$$

Similarly, we could write:

$$\mathcal{L}(y'''(t)) = s^3Y(s) - s^2y(0) - sy'(0) - y''(0)$$

And so on. These are special cases of Table Entry 18 (from the table in the book).

- Use Laplace transforms to solve the following IVP:

$$y'' + 2y' + 5y = 0, \quad y(0) = 2, y'(0) = -1$$

SOLUTION:

$$\mathcal{L}(y'') + 2\mathcal{L}(y') + 5\mathcal{L}(y) = 0$$

$$(s^2Y - 2s + 1) + 2(sY - 2) + 5Y = 0 \quad \Rightarrow \quad Y = \frac{2s+3}{s^2+2s+5}$$

To solve the ODE, we invert the transform (complete the square in the denominator):

$$Y = \frac{2s+3}{(s^2+2s+1)+4}$$

This expression may seem familiar; we inverted it in the last video:

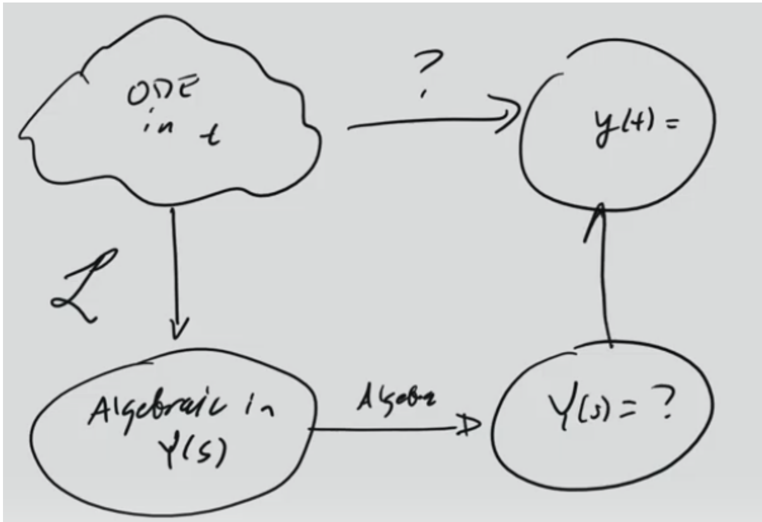
$$\frac{2(s+1)+1}{(s+1)^2+2^2} = 2\frac{s+1}{(s+1)^2+2^2} + \frac{1}{2}\frac{2}{(s+1)^2+2^2}$$

Now we can do the inversion:

$$y(t) = 2\mathrm{e}^{-t}\cos(2t) + \frac{1}{2}\mathrm{e}^{-t}\sin(2t)$$

And we also might note that this is the homogeneous solution to the differential equation (as expected).

The technique of solving a differential equation via a Laplace transform can be visualized using the diagram below.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