HW to Replace 7.1: Systems of Differential Equations

Try them all, but turn in solutions to 1(a), 3(a) and all of 6.

1. For each second order differential equation below, write the corresponding system of first order equations. Lastly, write the system in matrix-vector form.

   (a) $y'' + 2y' - 3y = 0$

   (b) $y'' + 4y' + 4y = 0$

   (c) $y'' - 9y = 0$

   (d) $y'' - 2y' + 2y = 0$

2. For extra practice, go back and give the general solution to each second order DE (using methods from Chapter 3 of our textbook).

3. For each system of first order, convert to a corresponding second order differential equation.

   (a) $\begin{aligned} x' &= 3x - 2y \\ y' &= 2x - 2y \end{aligned}$ (b) $\begin{aligned} x' &= -2x + y \\ y' &= x - 2y \end{aligned}$ (c) $\begin{aligned} x' &= x + y \\ y' &= 4x - 2y \end{aligned}$

4. For extra practice, solve each of the second order DEs from the previous exercise using techniques from Chapter 3 of our text.

5. Consider the system below:

   $$\begin{aligned} x' &= -3x + y \\ y' &= -2y \end{aligned}$$

   Solve this system by recognizing that we can solve for y directly, then substitute this into the DE for x and solve it as a first order linear DE.

6. Each system below is *nonlinear*. Solve each by first writing the system as dy/dx.

   (a) $\begin{aligned} x' &= y(1 + x^3) \\ y' &= x^2 \end{aligned}$ (b) $\begin{aligned} x' &= 4 + y^3 \\ y' &= 4x - x^3 \end{aligned}$ (c) $\begin{aligned} x' &= 2x^2y + 2x \\ y' &= -(2xy^2 + 2y) \end{aligned}$

7. Convert the third order (nonlinear) differential equation into a system of first order equations.

   $$y''' - y'' + y'y = t^2$$