

# Study Guide: Exam 1, Math 244

The exam covers material from Chapters 1 and 2 (up to 2.6), and will be 50 minutes in length. You may not use the text, notes, colleagues or a calculator.

One way to think about a differential equation is that it defines a set of functions (the solution). We can understand functions several ways- Graphically, Algebraically, and Numerically. In Chapters 1 and 2, we get a little of the first and third, and a lot of the second.

In summary, the first exam is all about understanding (and solving) first order differential equations:  $y' = f(t, y)$ .

## Vocabulary

- You should know what these terms mean:

differential equation, ordinary differential equation, partial differential equation, order of a differential equation, linear differential equation, equilibrium solution, isocline, direction field

- Be able to identify the following types of DEs: Linear, separable, autonomous.
- We discussed homogeneous first order DEs and Bernoulli DEs very briefly. In these cases, I will give you a suggested substitution. For example, in the homogeneous case, I will say: "Let  $y = v(x)/x$ , and show...".

## Direction Fields

Be able to match direction fields to their corresponding DEs. Be able to identify a differential equation of a special type by its direction field- in particular,  $y' = f(t)$  and  $y' = f(y)$ .

Be able to use a direction field to analyze the behavior of solutions to general first order equations. Be able to construct simple direction fields using isoclines. Be able to determine which DE best matches a given direction field.

## The Existence and Uniqueness Theorem

We had the two Existence and Uniqueness Theorems (one for linear, one for everything else):

Suppose we are given the IVP  $y' + p(t)y = g(t)$  at  $(t_0, y_0)$ . If  $p, g$  are continuous on an interval  $I$  that contains  $t_0$ , then there exists a unique solution to the initial value problem and that solution is valid for all  $t$  in the interval  $I$

Suppose we are given the IVP  $y' = f(t, y)$  with  $y(t_0) = y_0$ . If  $f$  and  $\partial f/\partial y$  are continuous in an open rectangle  $R$  containing the initial point  $(t_0, y_0)$ , then there exists a unique solution to the IVP. To find the interval on which the solution is valid, we would need to solve the IVP and look for restrictions on  $y(t)$ .

## Autonomous DEs:

The main idea here is to be able to graph the phase plot,  $y' = f(y)$  in the  $(y, y')$  plane and be able to translate the information from this graph to the direction field, the  $(t, y)$  plane.

Here is a summary of that information:

In Phase Diagram:	In Direction Field:
$y$ intercepts	Equilibrium Solutions
+ to - crossing	Stable Equilibrium
- to + crossing	Unstable Equilibrium
$y' > 0$	$y$ increasing
$y' < 0$	$y$ decreasing
$y'$ and $df/dy$ same sign	$y$ is concave up
$y'$ and $df/dy$ mixed	$y$ is concave down

Recall that we also looked at a theorem about determining the stability of an equilibrium solution using the sign of  $df/dy$ , and determining a formula for  $y''$  given  $y' = f(y)$ .

## Analytic Solutions

- Linear:  $y' + p(t)y = g(t)$ . Use the integrating factor:  $e^{\int p(t) dt}$
- Separable:  $y' = f(y)g(t)$ . Separate variables:  $(1/f(y)) dy = g(t) dt$
- Solve by substitution (as discussed, the substitutions will be provided)
  - Homogeneous:  $\frac{dy}{dx} = F(y/x)$ . Substitute  $v = y/x$  (and get the expression for  $dv/dx$  as well).
  - Bernoulli:  $y' + p(t)y = g(t)y^n$  Divide by  $y^n$ , let  $w = y^{1-n}$  and it becomes linear.
- Exact:  $M(x, y) + N(x, y)\frac{dy}{dx}$ , where  $N_x = M_y$ .  
 Solution: Set  $f_x(x, y) = M(x, y)$ . Integrate w/r to  $x$ . Check that  $f_y = N(x, y)$ , and add a function of  $y$  if necessary.

## Models

Be familiar with (be able to construct) the following models:

Exponential growth, Logistic growth, Falling body, Newton's Law of Cooling, Tank Mixing, and compound interest.