Worksheet: Linear Operators

Summary:

An operator F is said to be a linear operator if two things are true:
"For every function f, g and constant c:

$$F(f+g) = F(f) + F(g) \qquad \text{and} \qquad F(cf) = cF(f)$$

Often these are combined into one statement (which is fine if you want to do this):
"For every function f, g and constants c, d:

$$F(cf + dg) = c\,F(f) + d\,F(g)$$

The operator associated with a differential equation (in y): Get all terms with y on the left side of the equation, and everything else on the right. What you have on the left side is the operator acting on y.

Practice with operators

1. Let $R(f)$ be the operator defined by: $R(f) = f''(t) + 3t^2 f(t)$. Find $R(f)$ for each function below:

   (a) $f(t) = t^2$
   (b) $f(t) = \sin(3t)$
   (c) $f(t) = 2t - 5$

2. Let R be the operator defined in the previous problem. Show that R is a linear operator.

3. Let $F(y) = y'' + y - 5$. Explain why F is not linear.

4. Find the operator associated with the given differential equation, and classify it as linear or not linear:

   (a) $y' = ty^2 + \cos(t)$
   (b) $y'' = 4y' + 3y + \sin(t)$
   (c) $y' = \mathrm{e}^t y + 5$
   (d) $y'' = -\cos(y) + \cos(t)$