Study Guide: Exam 1, Math 244

The exam covers material from Chapters 1 and 2 (up to 2.6), and will be 50 minutes in length. You may not use the text, notes, colleagues or a calculator.

One way to think about a differential equation is that it defines a set of functions (the solution). We can understand functions several ways- Graphically, Algebraically, and Numerically. In Chapters 1 and 2, we get a little of the first and third, and a lot of the second.

In summary, the first exam is all about understanding (and solving) first order differential equations: $y' = f(t, y)$.

Vocabulary

- You should know what these terms mean:

  differential equation, ordinary differential equation, partial differential equation, order of a differential equation, linear differential equation, equilibrium solution, isocline, direction field

- Be able to identify the following types of DEs: Linear, separable, autonomous, exact.
- We discussed homogeneous first order DEs and Bernoulli DEs very briefly. In these cases, I will give you a suggested substitution. For example, in the homogeneous case, I will say: "Let $y = v(x)/x$, and show...".

Direction Fields

Be able to match direction fields to their corresponding DEs. Be able to identify a differential equation of a special type by its direction field- in particular, $y' = f(t)$ and $y' = f(y)$.

Be able to use a direction field to analyze the behavior of solutions to general first order equations. Be able to construct simple direction fields using isoclines. Be able to determine which DE best matches a given direction field.

The Existence and Uniqueness Theorem

We had the two Existence and Uniqueness Theorems (one for linear, one for everything else):

Suppose we are given the IVP $y' + p(t)y = g(t)$ at (t_0, y_0). If p, g are continuous on an interval I that contains t_0, then there exists a unique solution to the initial value problem and that solution is valid for all t in the interval I

Suppose we are given the IVP $y' = f(t, y)$ with $y(t_0) = y_0$. If f and $\partial f/partialy$ are continuous in an open rectangle R containing the initial point (t_0, y_0), then there exists a unique solution to the IVP. To find the interval on which the solution is valid, we would need to solve the IVP and look for restrictions on $y(t)$.

Autonomous DEs:

The main idea here is to be able to graph the phase plot, $y' = f(y)$ in the (y, y') plane and be able to translate the information from this graph to the direction field, the (t, y) plane.

Here is a summary of that information:

In Phase Diagram:	In Direction Field:
y intercepts	Equilibrium Solutions
$+$ to $-$ crossing	Stable Equilibrium
$-$ to $+$ crossing	Unstable Equilibrium
$y' > 0$	y increasing
$y' < 0$	y decreasing
y' and df/dy same sign	y is concave up
y' and df/dy mixed	y is concave down

Recall that we also looked at a theorem about determining the stability of an equilibrium solution using the sign of df/dy, and determining a formula for y'' given $y' = f(y)$.

Analytic Solutions

- Linear: $y' + p(t)y = g(t)$. Use the integrating factor: $e^{\int p(t)\,dt}$
- Separable: $y' = f(y)g(t)$. Separate variables: $(1/f(y))\,dy = g(t)\,dt$
- Solve by substitution (as discussed, the substitutions will be provided)
  - Homogeneous: $\frac{dy}{dx} = F(y/x)$. Substitute $v = y/x$ (and get the expression for dv/dx as well).
  - Bernoulli: $y' + p(t)y = g(t)y^n$ Divide by y^n, let $w = y^{1-n}$ and it becomes linear.
- Exact: $M(x,y) + N(x,y)\frac{dy}{dx}$, where $N_x = M_y$.

  Solution: Set $f_x(x,y) = M(x,y)$. Integrate w/r to x. Check that $f_y = N(x,y)$, and add a function of y if necessary.

Models

Be familiar with (be able to construct) the following models:

Exponential growth, Logistic growth, Falling body, Newton's Law of Cooling, Tank Mixing, and compound interest (with continuous compounding).