Review Problems, Exam 2, ODE

1. Solve using the method of undetermined coefficients:

$$y'' + 3y' + 2y = u_2(t), \quad y(0) = 0,\ y'(0) = 0$$

2. Solve the previous problem using Laplace transforms. To compare solutions, you should not use hyperbolic sine/cosine (so do a full partial fraction decomposition where appropriate).

3. Let $y'' + ky' + \omega^2 y = A\cos(\omega_0 t)$. What is the physical interpretation of k if this models a pendulum or a spring? Under what conditions on k, ω will the solution to the differential equation exhibit *beating*? *resonance*?

4. Solve using the method of Variation of Parameters, then check your answer using the method of undetermined coefficients:

$$3y'' - 3y' - 6y = 6\mathrm{e}^{-t}$$

5. The Variation of Parameters method is useful is describing the solution to a differential equation with a generic forcing function $g(t)$. Give the particular solution using this method to:

$$y'' - 5y' + 6y = g(t)$$

and rewrite your answer as a single integral.

6. Suppose that the solution to a second order linear homogeneous differential equation with constant coefficients is:

$$y_h(t) = C_1\mathrm{e}^{-2t} + c_2 t\mathrm{e}^{-2t}$$

If the forcing function is given to by the expression to the left of the table, write in the ansatz for the particular solution in the column to the right:

Forcing Function	Give final ansatz for $y_p(t)$
$3t^2 - 1$	
$5\mathrm{e}^t + 2\mathrm{e}^{-2t}$	
$t\sin(3t)$	

7. True or False, and explain: The solution to every second order differential equation can be written as: $y(t) = y_h(t) + y_p(t)$, where $y_h(t)$ is the solution to the homogeneous equation, and $y_p(t)$ is the particular solution.

8. Let $y'' - xy' - y = 0$. Find the recurrence relation between the coefficients of the power series solution based at (i) $x_0 = 0$ and (ii) $x_0 = 1$.

9. Find the first five coefficients of the power series solution to $(4 - x^2)y'' + 2y = 0$ if $y(0) = 0$ and $y'(0) = 1$. Write the solution up to 5th order using your answer.

10. (a) Rewrite the following as a single sum whose generic term involves x^n:

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x_n$$

(b) If this sum must be zero for all x, what relation between the a_n must hold? HINT: If $\sum c_n x^n = 0$ for all x, then $c_n = 0$ for all n.

11. If $y(x) = \sum_{n=0}^{\infty} a_n x^n$, substitute this into the differential equation in Problem 8. Rewrite the expression as a single sum equal to zero, and solve for the recurrence relation at $x_0 = 0$. You'll use the technique from Problem 10.