Practice Sheet: Complex Functions in Differential Equations

1. Solve for λ:

   (a) $\lambda^2 - 6\lambda + 13 = 0$

   (b) $2\lambda^3 - 8\lambda^2 + 12\lambda - 8 = 0$ (Hint: One solution is $\lambda = 2$).

   (c) $\lambda^3 + \lambda^2 + 4\lambda + 4 = 0$ (Hint: Try to factor in groups)

   (d) $\lambda^4 - 18\lambda^2 + 81 = 0$

2. Simplify each of the following to the form $a + bi$:

   (a) $(3 + 5i)(2 - 3i)$

   (b) $(3 - 2i)\overline{3 - 2i}$

   (c) $|-2 + i|$

   (d) $(2 - i)/(1 + 3i)$

   (e) $(1 + i)^{2+3i}$

   (f) $\ln\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

   (g) $e^{2+i}e^{-3i}$

   (h) $\ln(-5)$

   (i) $e^{-i\pi}$

3. Write each complex number in its polar form:

   (a) $1 - \sqrt{3}i$

   (b) 3

   (c) $-5 - 3i$ (Use a calculator for θ)

   (d) $-i$

4. Show, using Euler's Formula and treating i as a constant, that:

$$\frac{d}{dt}\left(e^{(\alpha+\beta i)t}\right) = (\alpha + \beta i)e^{(\alpha+\beta i)t}$$

5. Verify our in-class remarks that, if $y = C_1e^{(\alpha+\beta i)t} + C_2e^{(\alpha-\beta i)t}$, and $y(0) = 1$, $y'(0) = \alpha$, then $y = e^{\alpha t}\cos(\beta t)$.

6. Similarly, verify that, if $y = C_1e^{(\alpha+\beta i)t} + C_2e^{(\alpha-\beta i)t}$, and $y(0) = 0$, $y'(0) = \beta$, then $y = e^{\alpha t}\sin(\beta t)$.