REVIEW QUESTIONS, EXAM 2, Math 244

1. True or False, and explain:
   (a) Let f and g be differentiable for every x. If the $W(f,g)=0$ for every x, f,g must be linearly dependent.
   (b) We cannot draw a direction field for a second order differential equation.
   (c) Given that y_1 is part of the homogeneous solution, we can find both y_2 and y_p (at the same time) to $ay''+by'+cy=f(x)$.
   (d) We can always compute a fundamental set of solutions to $ay''+by'+cy=0$.
   (e) The Cauchy-Euler equation, $ax^2y''+bxy'+cy=0$ can be written as $a\hat{y}''+b\hat{y}'+c\hat{y}=0$ after an appropriate substitution (if True, write the substitution).
   (f) In using the Method of Undetermined Coefficients, is the ansatz $y_p=(Ax^2+Bx+C)(D\sin(x)+E\cos(x))$ equivalent to

$$y_p=(Ax^2+Bx+C)\sin(x)+(Dx^2+Ex+F)\cos(x)$$

2. Suppose $W(f,g)=t^2-4$. What can we conclude about f,g?

3. Without using the Wronskian, determine whether the given set of functions is linearly independent on the indicated interval:
   (a) $\ln(x)$, $\ln(x^2)$, $(0,\infty)$
   (b) x, $x+1$, $(-\infty,\infty)$
   (c) $xe^{x+1}, (4x-5)e^x, xe^x$, $(-\infty,\infty)$.

4. Finish the definition: Functions f,g are said to be Linearly Independent on an interval I if:

5. One way to obtain a fundamental set of solutions was to solve two initial value problems. How did we show that $y_1=e^{\alpha x}\cos(\beta x)$, $y_2=e^{\alpha x}\sin(\beta x)$ formed a fundamental set when $m=\alpha\pm\beta i$ was the solution to the characteristic equation?

6. Suppose $m_1=3, m_2=-5, m_3=1$ are the roots of multiplicity one, two and three respectively, of the characteristic equation. Write the general solution of the corresponding homogeneous linear DE if it is (a) an equation with constant coefficients, (b) a Cauchy-Euler equation.

7. Write down the general solution of the given differential equation using Method of Undetermined Coefficients (There will be two cases, $\omega=\alpha$ and $\omega\neq\alpha$). Do not solve for the coefficients: (a) $y''+\omega^2y=\sin(\alpha x)$, (b) $y''-\omega^2y=e^{\alpha x}$.

8. Find a linear second order differential equation with constant coefficients for which $y_1 = 1$ and $y_2 = \mathrm{e}^{-x}$ are solutions to the homogeneous equation, and $y_p = \frac{1}{2}x^2 - x$ is a particular solution.

9. Write the solution in terms of α, then determine the value(s) of α so that $y(t) \to 0$ as $t \to \infty$:

$$y'' - y' - 6y = 0, \qquad y(0) = 1, y'(0) = \alpha$$

10. Determine the longest interval for which the IVP is certain to have a unique solution. Do not attempt to find the solution:

$$t(t-4)y'' + 3ty' + 4y = 2, \quad y(3) = 0, y'(3) = -1$$

11. Let $L(y) = y'' - 6y' + 5y$. Suppose that $y_{p_1} = 3\mathrm{e}^{2x}$ and $y_{p_2} = x^2 + 3x$ are particular solutions to $L(y) = -9\mathrm{e}^{2x}$ and $L(y) = 5x^2 + 3x - 16$. What is the particular solution to

$$y'' - 6y' + 5y = -10x^2 - 6x + 32 + \mathrm{e}^{2x}$$

12. Below you are given a differential equation and one of the homogeneous solutions. Use reduction of order to either find the other homogeneous solution, or both the homogeneous and particular solutions:

    (a) $(1 - x^2)y'' + 2xy' = 0$, $y_1 = 1$

    (b) $y'' - 3y' + 2y = 5\mathrm{e}^{3x}$, $y_1 = \mathrm{e}^x$

    (c) $4x^2y'' + y = 0$, $y_1 = x^{1/2}\ln(x)$

13. Referring to the previous problem, solve (a) by using a suitable substitution to make it first order, solve (b) by the Method of Undetermined Coefficents, and (c) by seeing it is a Cauchy-Euler equation.

14. Solve. If given an IVP, solve for all unknown coefficients.

    (a) $y'' + 2y' + y = 0, y(0) = 1, y'(0) = 0$

    (b) $y'' - y = x + \sin(x)$, $y(0) = 2$, $y'(0) = -3$

    (c) $y'' - y = \frac{2\mathrm{e}^x}{\mathrm{e}^x + \mathrm{e}^{-x}}$

    (d) $y'' - 2y' + y = x^2\mathrm{e}^x$

    (e) $x^2y'' - xy' + y = x^3$

    (f) $x^3y''' - 6y = 0$

    (g) $2x^2y'' + 5xy' + y = x^2 - x$

    (h) Solve by Variation of Parameters: $2y'' + y' - y = x + 1$

15. Be sure you can do the homework from Section 4.8!