

Overview of Math 244: Spr 05

For the exam, I will provide the table of Laplace transforms, the Poincare Diagram, the formulas for Reduction of Order/Variation of Parameters:

$$y_2 = y_1(x) \int \frac{e^{-\int P(x) dx}}{y_1^2(x)} dx \quad u'_1 = \frac{-y_2 f(x)}{W}, \quad u'_2 = \frac{y_1 f(x)}{W}$$

and will give you integrating factors or suggested substitutions (e.g., exact with integrating factor (p. 71), the other homogeneous (2.5), etc). For review, go back through the exam review sheets and old exams and quizzes. For the new material, be sure you can do the homework and the problems from the last handout.

Methods for Solving First Order Equations:

$$\begin{aligned} y' &= f(t)g(y) && \text{Separable} \\ y' + p(t)y &= g(t) && \text{Integrating Factor} \\ N dx + M dy &= 0 && \text{Exact} \end{aligned}$$

Second Order Equations:

DE	Ansatz	Notes
$ay'' + by' + cy = f(t)$	$y_h(t) = e^{\lambda t}$	y_p Var of Param Undet Coeffs
$\mathbf{x}' = A\mathbf{x}$	$\mathbf{x}(t) = \mathbf{v}e^{\lambda t}$	Elimination (4.8)
$ax^2y'' + bxy' + cy = 0$	$y_h = x^r$	Cauchy-Euler Three cases for r
$y'' + p(t)y' + q(t)y = f(t)$	$Y(s) = \mathcal{L}(y(t))$	$y(t)$ Exp Order and PC
$y'' + p(x)y' + q(x)y = f(x)$		
x_0 is ordinary pt	$y(x) = \sum c_n(x - x_0)^n$	
x_0 is reg sing pt	$y(x) = \sum c_n(x - x_0)^{n+r}$	

Also recall:

- Definition of linear operator, a linear D.E. (Theory, Sect 4.1)
- Linear independence and Wronskian
- Integration by parts, and partial fractions
- Modeling expressions (exp growth, logistic growth, proportional, interaction term), Tank mixing
- Resonance, over-, under- and critically damped (5.1)
- Autonomous First Order: Find and classify equilibria, draw phase plot and direction field. Where is y inc/dec, concave up/down.