Integral Practice, Part II SOLUTIONS

1. $\int x^{1/3}(x-1)\,dx \int x^{4/3} - x^{1/3}\,dx = \frac{3}{7}x^{7/3} - \frac{3}{4}x^{4/3} + C$

2. Do long division first, so: $\frac{x^3+1}{x^2+1} = x - \frac{x}{x^2+1} + \frac{1}{x^2+1}$. Integrating, we get: $\frac{1}{2}x^2 - \frac{1}{2}\ln(x^2+1) + \tan^{-1}(x) + C$

3. Do u, du substitution ($u = x^2 - 1, du = 2x\,dx$):
$$\frac{1}{2}\int u^{-2}\,du = -\frac{1}{2}(x^2-1)^{-1} + C$$

4. Do integration by parts:
$$\begin{array}{ccc} + & x^3 & \mathrm{e}^x \\ - & 3x^2 & \mathrm{e}^x \\ + & 6x & \mathrm{e}^x \\ - & 6 & \mathrm{e}^x \\ + & 0 & \mathrm{e}^x \end{array} \Rightarrow \mathrm{e}^x\left(x^3 - 3x^2 + 6x - 6\right) + C$$

5. Integration by parts:
$$\begin{array}{ccc} + & \ln(t) & t^{1/2} \\ - & \frac{1}{t} & \frac{2}{3}t^{3/2} \end{array} \Rightarrow \int \sqrt{t}\ln(t)\,dt = \frac{2}{3}t^{3/2}\ln(t) - \frac{2}{3}\int t^{1/2}\,dt = \frac{2}{3}t^{3/2}\ln(t) - \frac{4}{9}t^{3/2} + C$$

6. $\int \sec^2(x)\,dx = \tan(x) + C$

7. $\int \cos^2(2x)\,dx = \frac{1}{2}\int (1+\cos(4x))\,dx = \frac{1}{2}\left(x + \frac{1}{4}\sin(4x)\right) + C$

8. $\int x\sqrt{16-3x}\,dx$. Let $u = 16-3x$, and $du = -3\,dx$. Also substitute $x = -\frac{1}{3}(u-16)$ so that
$$\int x\sqrt{16-3x}\,dx = \frac{1}{9}\int (u-16)u^{1/2}\,du = \frac{1}{9}\int u^{3/2} - 16u^{1/2}\,du =$$
$$\frac{1}{9}\left(\frac{2}{5}(16-3x)^{5/2} - \frac{32}{3}(16-3x)^{3/2}\right) + C$$

9. $\int \mathrm{e}^{2t}\sin(3t)\,dt$ Use integration by parts twice:
$$\begin{array}{ccc} + & \sin(3t) & \mathrm{e}^{2t} \\ - & 3\cos(3t) & \frac{1}{2}\mathrm{e}^{2t} \\ + & -9\sin(3t) & \frac{1}{4}\mathrm{e}^{2t} \end{array} \Rightarrow \int \mathrm{e}^{2t}\sin(3t)\,dt = \mathrm{e}^{2t}\left(\frac{1}{2}\sin(3t) - \frac{3}{4}\cos(3t)\right) - \frac{9}{4}\int \mathrm{e}^{2t}\sin(3t)\,dt$$
$$\frac{13}{4}\int \mathrm{e}^{2t}\sin(3t)\,dt = \mathrm{e}^{2t}\left(\frac{1}{2}\sin(3t) - \frac{3}{4}\cos(3t)\right) \Rightarrow \mathrm{e}^{2t}\left(\frac{2}{13}\sin(3t) - \frac{3}{13}\cos(3t)\right) + C$$

10. Simplify to get: $\int x^{3/2} - x^{1/2} + x^{-1/2}\,dx = \frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + 2x^{1/2} + C$

11. Let $u = \ln(x)$, or $x = \mathrm{e}^u$ so that $dx = \mathrm{e}^u\,du$.

$$\int x^4(\ln(x))^2\,dx = \int \mathrm{e}^{4u}u^2\mathrm{e}^u\,du = \int u^2\mathrm{e}^{5u}\,du$$

Integration by parts:

$$\begin{array}{ccc} + & u^2 & \mathrm{e}^{5u} \\ - & 2u & \frac{1}{5}\mathrm{e}^{5u} \\ + & 2 & \frac{1}{25}\mathrm{e}^{5u} \\ - & 0 & \frac{1}{125}\mathrm{e}^{5u} \end{array} \Rightarrow \mathrm{e}^{5u}\left(\frac{1}{5}u^2 - \frac{2}{25}u + \frac{2}{125}\right) = x^5\left(\frac{1}{5}\ln^2(x) - \frac{2}{25}\ln(x) + \frac{2}{125}\right) + C$$

12. Integration by parts:

$$\begin{array}{ccc} + & \sin^{-1}(x) & 1 \\ - & \frac{1}{\sqrt{1-x^2}} & x \end{array} \Rightarrow \int \sin^{-1}(x)\,dx = x\sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx = x\sin^{-1}(x) + \sqrt{1-x^2} + C$$

(Use $u = 1 - x^2$, $du = -2\,dx$ for the last integration)

13. $\int \frac{y}{y+2}\,dy = \int 1 - \frac{2}{y+2}\,dy$ You could get this expression either through long division, or by writing: $\frac{y}{y+2} = \frac{y+2-2}{y+2} = 1 - \frac{2}{y+2}$. The integral is therefore $y - 2\ln(y+2) + C$

14. $\int \frac{y^2+4y+4}{y}\,dy = \int y + 4 + \frac{4}{y}\,dy = \frac{1}{2}y^2 + 4y + 4\ln(y) + C$

15. $\int \frac{x-9}{(x+5)(x-2)}\,dx = \int \frac{2}{x+5} - \frac{1}{x-2}\,dx = 2\ln(x+5) - \ln(x-2) + C$

16. $\int \csc^2(3t)\,dt = \frac{1}{3}\cot(3t) + C$

17. $\int \frac{1}{x^2(x-1)(x+1)}\,dx = \int -\frac{1}{x^2} + \frac{1}{2(x-1)} - \frac{1}{2(x+1)}\,dx =$

$$\frac{1}{x} + \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x+1) + C$$