Solutions to Practice Problems:

1. $$\begin{aligned} C_1 + C_2 &= 2 \\ -2C_1 - 3C_2 &= 3 \end{aligned}$$

Solution:

$$C_1 = \frac{\begin{vmatrix} 2 & 1 \\ 3 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix}} = \frac{-6-3}{-3+2} = \frac{-9}{-1} = 9 \qquad C_2 = \frac{\begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix}}{-1} = \frac{3+4}{-1} = -7$$

2. $$\begin{aligned} C_1 + C_2 &= y_0 \\ r_1C_1 + r_2C_2 &= v_0 \end{aligned}$$

Solution:

$$C_1 = \frac{\begin{vmatrix} y_0 & 1 \\ v_0 & r_2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ r_1 & r_2 \end{vmatrix}} = \frac{y_0r_2 - v_0}{r_2 - r_1} \qquad C_2 = \frac{\begin{vmatrix} 1 & y_0 \\ r_1 & v_0 \end{vmatrix}}{r_2 - r_1} = \frac{v_0 - r_1y_1}{r_2 - r_1}$$

NOTE: A unique solution to the system of equations exists iff $r_1 \neq r_2$.

3. $$\begin{aligned} C_1 + C_2 &= 2 \\ 3C_1 + C_2 &= 1 \end{aligned}$$

Solution:

$$C_1 = \frac{\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}} = \frac{1}{-2} = -\frac{1}{2} \qquad C_2 = \frac{\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}}{-2} = \frac{-5}{-2} = \frac{5}{2}$$

4. $$\begin{aligned} 2C_1 - 5C2 &= 3 \\ 6C_1 - 15C_2 &= 10 \end{aligned}$$. No solution to this array (think of it as two parallel lines).

5. $$\begin{aligned} 2x - 3y &= -1 \\ 3x - 2y &= 1 \end{aligned}$$

Solution:

$$x = \frac{\begin{vmatrix} -1 & -3 \\ 1 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 3 & -2 \end{vmatrix}} = \frac{2+3}{-4+9} = 1 \qquad y = \frac{\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix}}{5} = \frac{2+3}{5} = 1$$