Hyperbolic Functions

In differential equations, the terms e^x and e^{-x} appear together so often, that it is convenient to define special functions that use them:

Definition: The hyperbolic sine, denoted by $\sinh(x)$, is defined as:

$$\sinh(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$$

Similarly, the hyperbolic cosine is defined:

$$\cosh(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$$

Practice questions with the hyperbolic functions:

1. Plot the hyperbolic sine and cosine. What do they look like? Are they periodic functions?

2. Show, using the definitions, that the hyperbolic sine is an odd function[1] and the hyperbolic cosine is even.

3. Show, using the definitions, that:

   $$\cosh^2(x) - \sinh^2(x) = 1$$

   (Don't confuse this with the Pythagorean Identity: $\cos^2(x) + \sin^2(x) = 1$)

4. Show, using the definitions, that:

   $$\frac{d}{dx}(\sinh(x)) = \cosh(x) \qquad \text{and} \qquad \frac{d}{dx}(\cosh(x)) = \sinh(x)$$

   (Don't confuse these with the derivatives of sine and cosine!)

5. Show that any function of the form:

   $$y = A\sinh(mt) + B\cosh(mt)$$

   satisfies the differential equation: $y'' = m^2 y$.

6. Show that any function of the form:

   $$y = A\sin(\omega t) + B\cos(\omega t)$$

   satisfies the differential equation: $y'' = -\omega^2 y$

[1] Recall that f is odd if $f(-x) = -f(x)$, and that f is even if $f(-x) = f(x)$.