M244
Spring 2007

Quiz 6

Name:_____

Instructions: This is a take home quiz, **due on Monday at the beginning of class.** You may use your text and class notes, but you are expected to do your own work. Write the solutions to the following on your own paper and attach it to this sheet. **Neatness will count!**

The symbol we will use for the Laplace transform is $\mathcal{L}$, and the table of Laplace transforms is on page 319.

1. Use the *definition* of the Laplace Transform to compute $\mathcal{L}(f(t))$, where $f(t)$ is given as:

$$f(t) = \begin{cases} t^2 & \text{if } 0 \le t < 1 \\ 2 + t & \text{if } t > 1 \end{cases}$$

   Be sure to include the details about the convergence of the improper integral (L'Hospital's rule might come in handy).

2. Problem 14, page 313. You may either use the formula given in the text, or the formula:

$$\mathrm{e}^{(a+ib)t} = \mathrm{e}^{at}\cos(bt) + i\mathrm{e}^{at}\sin(bt)$$

   and you may use Table Entry 2. Of course, you may NOT use Table Entry 10.

3. Find, using the table: $\mathcal{L}^{-1}\left(\dfrac{s}{s^2 + 2s - 3}\right)$

4. Find an expression for $Y(s)$, do not solve for $y(t)$:

$$y'' - 4y' + 4y = \mathrm{e}^t\cos(t), \quad y(0) = 0, y'(0) = 1$$

5. Solve the following IVP using the method of Laplace Transforms:

$$y'' + 3y' + 2y = \mathrm{e}^{-t} \qquad y(0) = 1 \quad y'(0) = 0$$