Exercise Set 2 (replaces Ch 7)

1. Verify that the following function solves the given system of DEs:

$$\mathbf{x}(t) = C_1 e^{-t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \mathbf{x}' = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} \mathbf{x}$$

2. Convert each of the systems $\mathbf{x}' = A\mathbf{x}$ into a single second order differential equation, and solve it using methods from Chapter 3:

   (a) $A = \begin{bmatrix} 1 & 2 \\ -5 & -1 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ (c) $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

3. Suppose we have a system of two differential equations, and the system gave us the eigenvalues and eigenvectors listed. For each, write the general solution to the differential equation:

   (a) $\lambda_1 = -2 \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \lambda_2 = -1 \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

   (b) $\lambda = -1 + i \quad \mathbf{v} = \begin{bmatrix} 1+i \\ 2 \end{bmatrix}$

   (c) $\lambda = 1, 1 \quad \mathbf{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ where $\mathbf{w}$ is the generalized eigenvector.

4. For each matrix, find the eigenvalues and eigenvectors. If there is only one eigenvector, find the associated generalized eigenvector.

   (a) $A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix}$

   (d) $A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$

5. For each system below, find y as a function of x by first writing the differential equation as dy/dx.

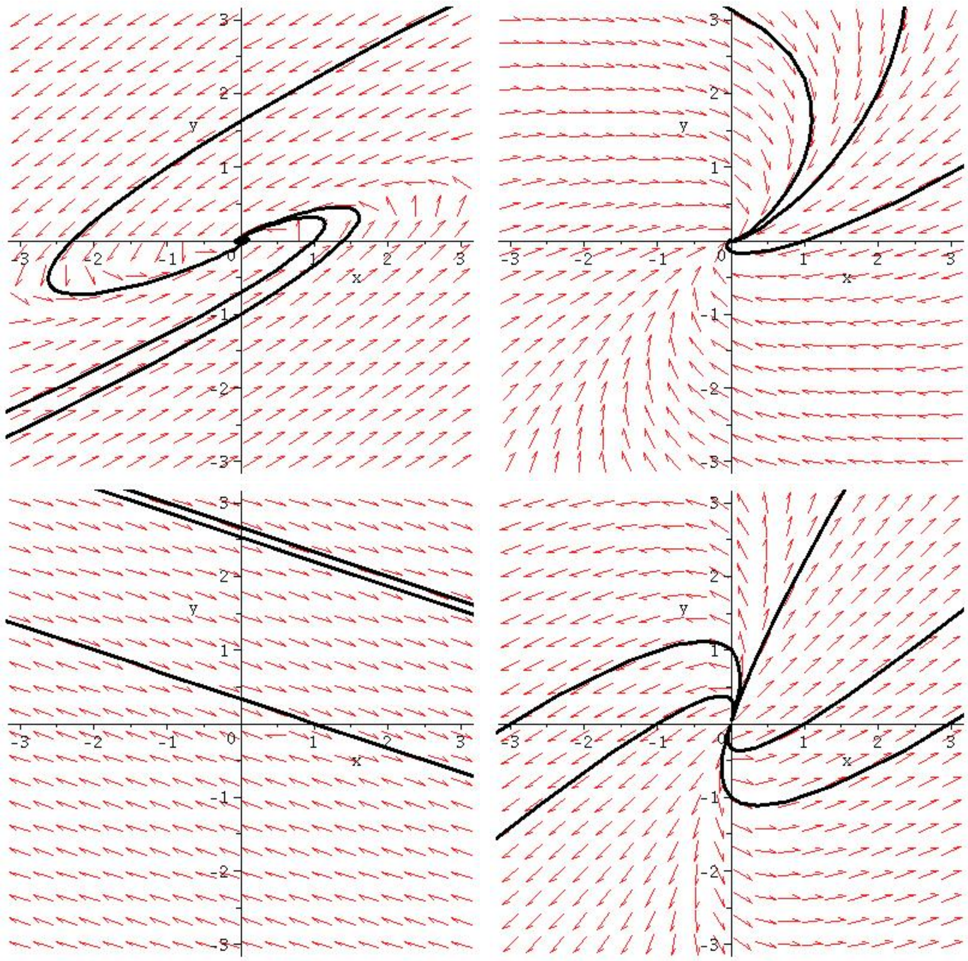
   (a) $\begin{aligned} x' &= -2x \\ y' &= y \end{aligned}$

   (b) $\begin{aligned} x' &= y + x^3 y \\ y' &= x^2 \end{aligned}$

   (c) $\begin{aligned} x' &= -(2x+3) \\ y' &= 2y - 2 \end{aligned}$

   (d) $\begin{aligned} x' &= -2y \\ y' &= 2x \end{aligned}$

   Side remark: These problems are good review for Chapter 2; and they represent a little different approach to solving the system (although these are special cases for which dy/dx are solvable).



6. Match the graphs in the figure to the system $\mathbf{x}' = A\mathbf{x}$, where A comes from Exercise 4 (a)-(d).