Graphs for Section 6.4 Homework

Problems 2:

```
> h:=Heaviside(t-Pi)-Heaviside(t-2*Pi);
```

$$h := \text{Heaviside}(t-\pi) - \text{Heaviside}(t-2\,\pi)$$

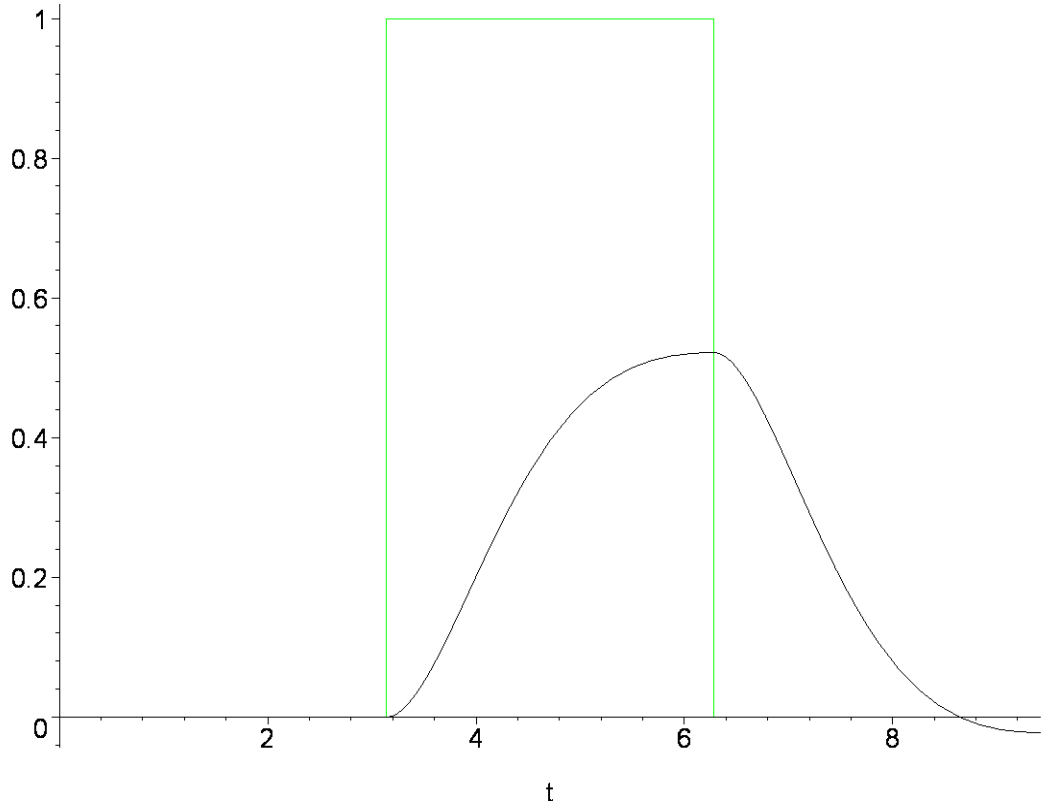
```
> DE02:=diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=h;
```

$$DE02 := \left(\frac{\partial^2}{\partial t^2}\,y(t)\right) + 2\left(\frac{\partial}{\partial t}\,y(t)\right) + 2\,y(t) = \text{Heaviside}(t-\pi) - \text{Heaviside}(t-2\,\pi)$$

```
> Y:=dsolve({DE02,y(0)=0,D(y)(0)=0},y(t),method=laplace);
```

$$Y := y(t) = \text{Heaviside}(t-\pi)\left(\frac{1}{2} + \frac{1}{2}\,\mathbf{e}^{(-t+\pi)}\cos(t) + \frac{1}{2}\,\mathbf{e}^{(-t+\pi)}\sin(t)\right)$$
$$+\left(-\frac{1}{2} + \frac{1}{2}\,\mathbf{e}^{(-t+2\,\pi)}\cos(t) + \frac{1}{2}\,\mathbf{e}^{(-t+2\,\pi)}\sin(t)\right)\text{Heaviside}(t-2\,\pi)$$

```
> plot([rhs(Y),h],t=0..3*Pi,color=[black, green]);
```



Problem 4

```
> h:=sin(t)+Heaviside(t-Pi)*sin(t-2*Pi);
```

$$h := \sin(t) + \text{Heaviside}(t-\pi)\sin(t)$$

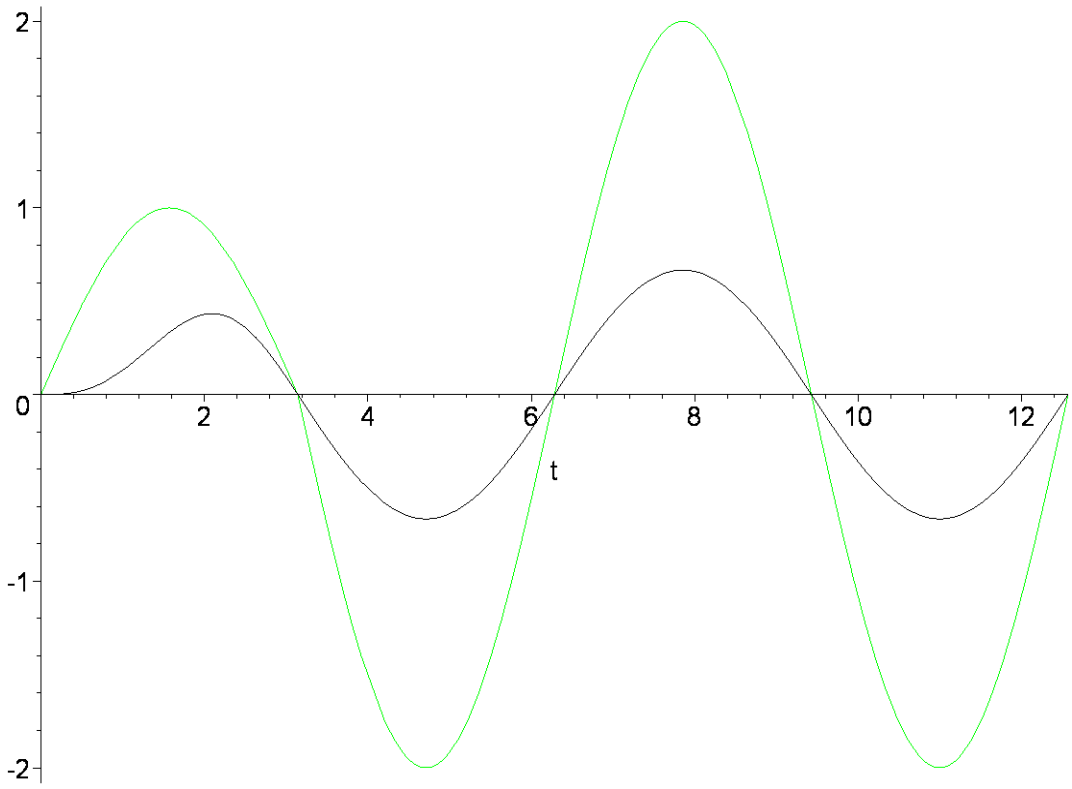
```
> DE04:=diff(y(t),t$2)+4*y(t)=h;
```

$$DE04 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 4\, y(t) = \sin(t) + \text{Heaviside}(t-\pi)\sin(t)$$

```
> Y:=dsolve({DE04,y(0)=0,D(y)(0)=0},y(t),method=laplace);
```

$$Y := y(t) = \left(\frac{1}{6}\sin(2t) + \frac{1}{3}\sin(t)\right)\text{Heaviside}(t-\pi) - \frac{1}{6}\sin(2t) + \frac{1}{3}\sin(t)$$

```
> plot([rhs(Y),h],t=0..4*Pi,color=[black, green]);
```



Problem 6

```
> h:=Heaviside(t-2);
```

$$h := \text{Heaviside}(t-2)$$

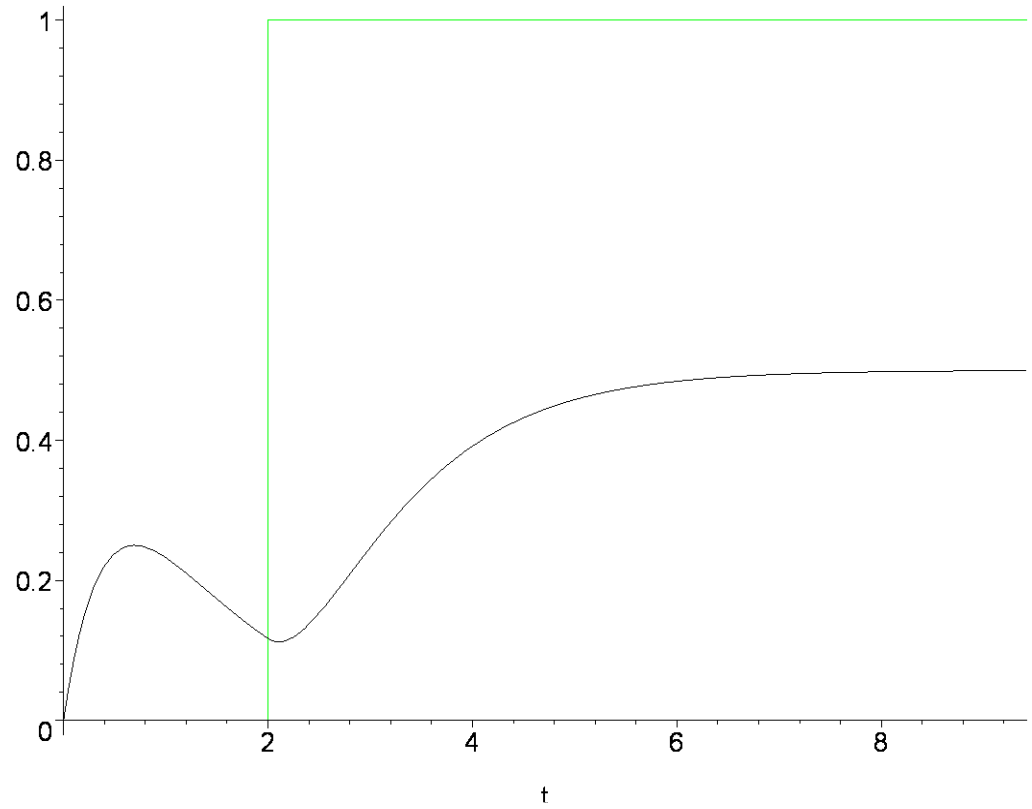
```
> DE06:=diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=h;
```

$$DE06 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 3\left(\frac{\partial}{\partial t} y(t)\right) + 2\, y(t) = \text{Heaviside}(t-2)$$

```
> Y:=dsolve({DE06,y(0)=0,D(y)(0)=1},y(t),method=laplace);
```

$$Y := y(t) = \text{Heaviside}(t-2)\left(\frac{1}{2} + \frac{1}{2}\mathbf{e}^{(-2t+4)} - \mathbf{e}^{(-t+2)}\right) + 2\,\mathbf{e}^{(-3/2\,t)}\sinh\left(\frac{1}{2}t\right)$$

```
> plot([rhs(Y),h],t=0..3*Pi,color=[black, green]);
```



Problem 9

```
> h:=(t/2)*(1-Heaviside(t-6))+3*Heaviside(t-6);
```

$$h := \frac{1}{2} t \,(1 - \text{Heaviside}(t-6)) + 3\,\text{Heaviside}(t-6)$$

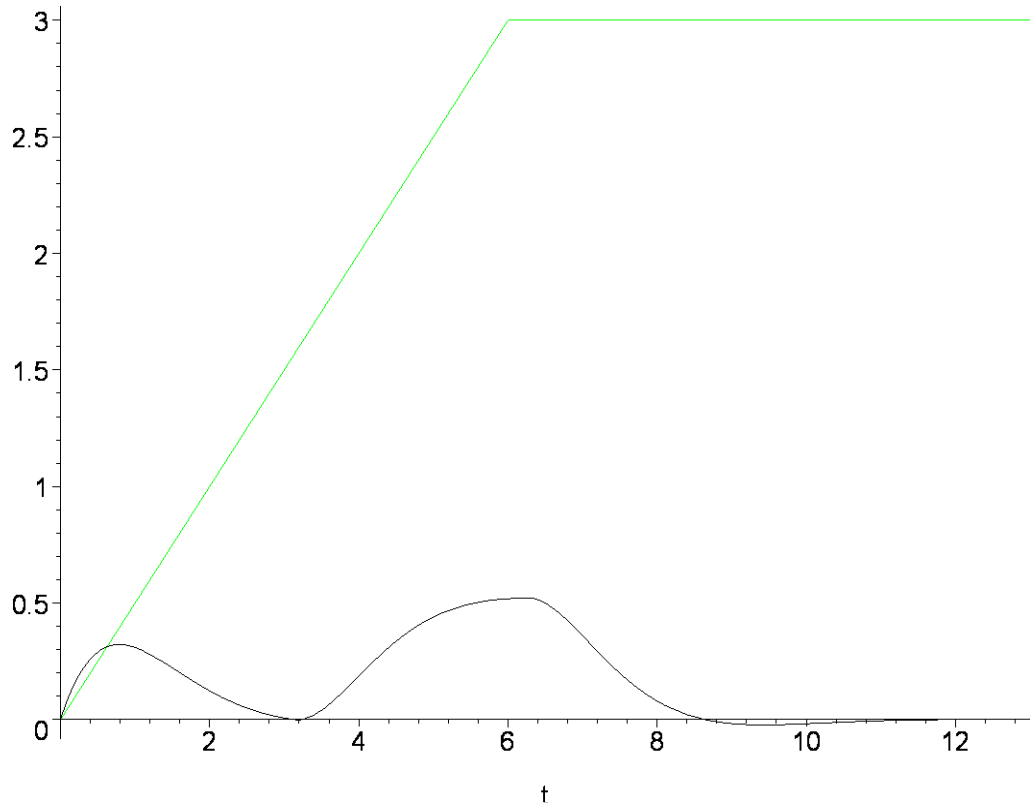
```
> DE09:=diff(y(t),t$2)+y(t)=h;
```

$$DE09 := \left(\frac{\partial^2}{\partial t^2} \mathrm{y}(t)\right) + \mathrm{y}(t) = \frac{1}{2} t \,(1 - \text{Heaviside}(t-6)) + 3\,\text{Heaviside}(t-6)$$

```
> Y:=dsolve({DE02,y(0)=0,D(y)(0)=1},y(t),method=laplace);
```

$$Y := \mathrm{y}(t) = \text{Heaviside}(t-\pi)\left(\frac{1}{2} + \frac{1}{2}\mathbf{e}^{(-t+\pi)}\cos(t) + \frac{1}{2}\mathbf{e}^{(-t+\pi)}\sin(t)\right)$$
$$+\left(-\frac{1}{2} + \frac{1}{2}\mathbf{e}^{(-t+2\pi)}\cos(t) + \frac{1}{2}\mathbf{e}^{(-t+2\pi)}\sin(t)\right)\text{Heaviside}(t-2\pi) + \mathbf{e}^{(-t)}\sin(t)$$

```
> plot([rhs(Y),h],t=0..13,color=[black, green]);
```

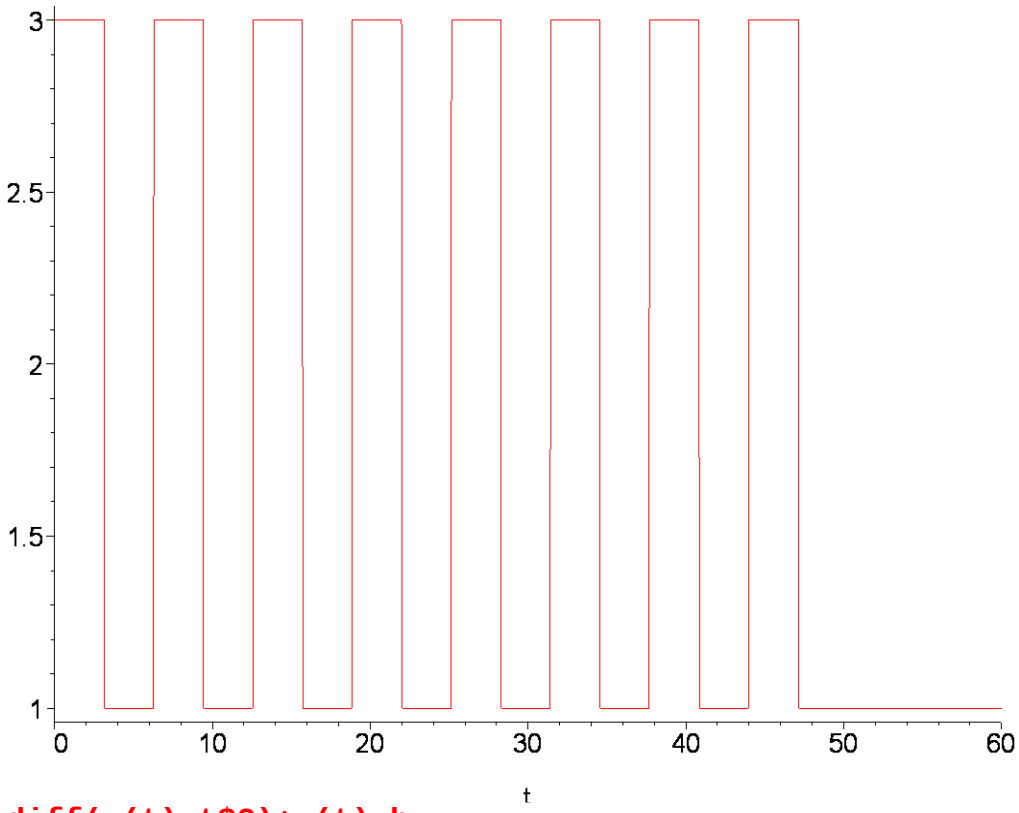


Problem 19

```
> h:=1+2*sum('((-1)^k)*Heaviside(t-k*Pi)', 'k'=0..15);
```

$$h := 1 + 2\,\mathrm{Heaviside}(t) - 2\,\mathrm{Heaviside}(t-\pi) + 2\,\mathrm{Heaviside}(t-2\pi) - 2\,\mathrm{Heaviside}(t-3\pi) + 2\,\mathrm{Heaviside}(t-4\pi) - 2\,\mathrm{Heaviside}(t-5\pi) + 2\,\mathrm{Heaviside}(t-6\pi) - 2\,\mathrm{Heaviside}(t-7\pi) + 2\,\mathrm{Heaviside}(t-8\pi) - 2\,\mathrm{Heaviside}(t-9\pi) + 2\,\mathrm{Heaviside}(t-10\pi) - 2\,\mathrm{Heaviside}(t-11\pi) + 2\,\mathrm{Heaviside}(t-12\pi) - 2\,\mathrm{Heaviside}(t-13\pi) + 2\,\mathrm{Heaviside}(t-14\pi) - 2\,\mathrm{Heaviside}(t-15\pi)$$

```
> plot(h,t=0..60);
```



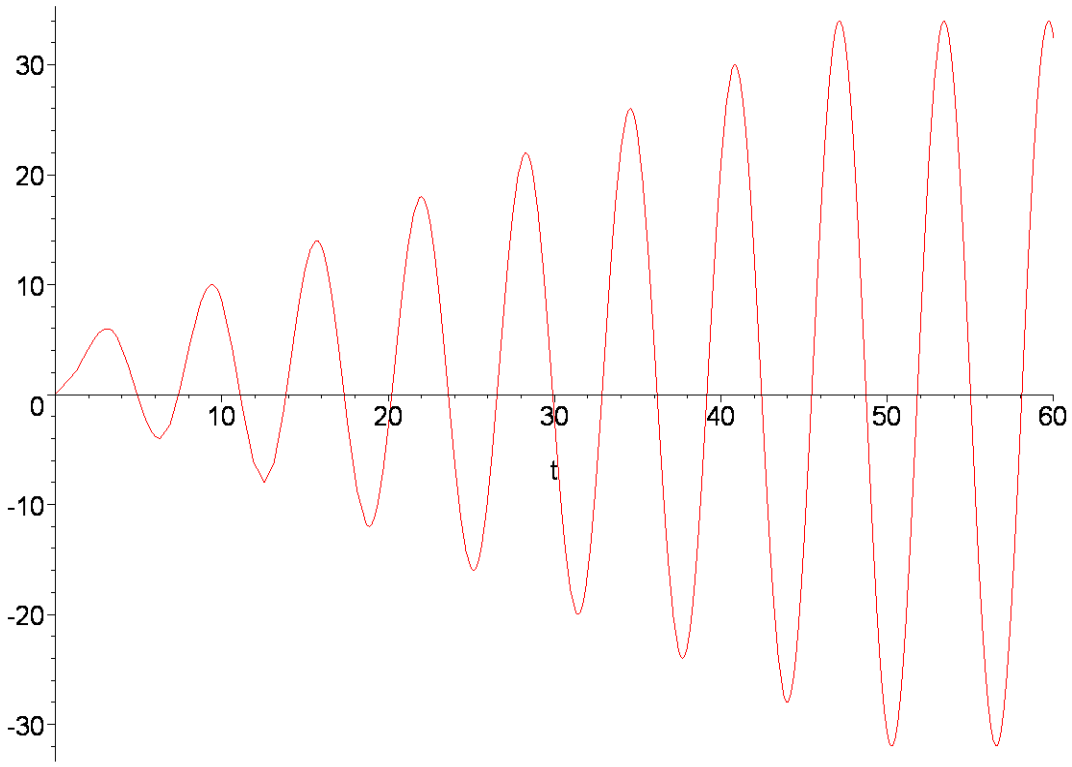

```
> DE19:=diff(y(t),t$2)+y(t)=h;
```

$$DE19 := \left(\frac{\partial^2}{\partial t^2}\,y(t)\right) + y(t) = 1 + 2\,\text{Heaviside}(t) - 2\,\text{Heaviside}(t-\pi) + 2\,\text{Heaviside}(t-2\pi) - 2\,\text{Heaviside}(t-3\pi) + 2\,\text{Heaviside}(t-4\pi) - 2\,\text{Heaviside}(t-5\pi) + 2\,\text{Heaviside}(t-6\pi) - 2\,\text{Heaviside}(t-7\pi) + 2\,\text{Heaviside}(t-8\pi) - 2\,\text{Heaviside}(t-9\pi) + 2\,\text{Heaviside}(t-10\pi) - 2\,\text{Heaviside}(t-11\pi) + 2\,\text{Heaviside}(t-12\pi) - 2\,\text{Heaviside}(t-13\pi) + 2\,\text{Heaviside}(t-14\pi) - 2\,\text{Heaviside}(t-15\pi)$$

```
> Y:=dsolve({DE19,y(0)=0,D(y)(0)=0},y(t),method=laplace);
```

$$Y := y(t) = 3 + (-4\,\text{Heaviside}(t-13\pi) - 4\,\text{Heaviside}(t-\pi) - 4\,\text{Heaviside}(t-15\pi) - 4\,\text{Heaviside}(t-3\pi) - 4\,\text{Heaviside}(t-5\pi) - 4\,\text{Heaviside}(t-7\pi) - 4\,\text{Heaviside}(t-9\pi) - 4\,\text{Heaviside}(t-11\pi))\cos\left(\frac{1}{2}t\right)^2 + (4\,\text{Heaviside}(t-4\pi) + 4\,\text{Heaviside}(t-6\pi) + 4\,\text{Heaviside}(t-8\pi) + 4\,\text{Heaviside}(t-10\pi) + 4\,\text{Heaviside}(t-12\pi) + 4\,\text{Heaviside}(t-14\pi) + 4\,\text{Heaviside}(t-2\pi))\sin\left(\frac{1}{2}t\right)^2 - 3\cos(t)$$

```
> plot(rhs(Y),t=0..60);
```

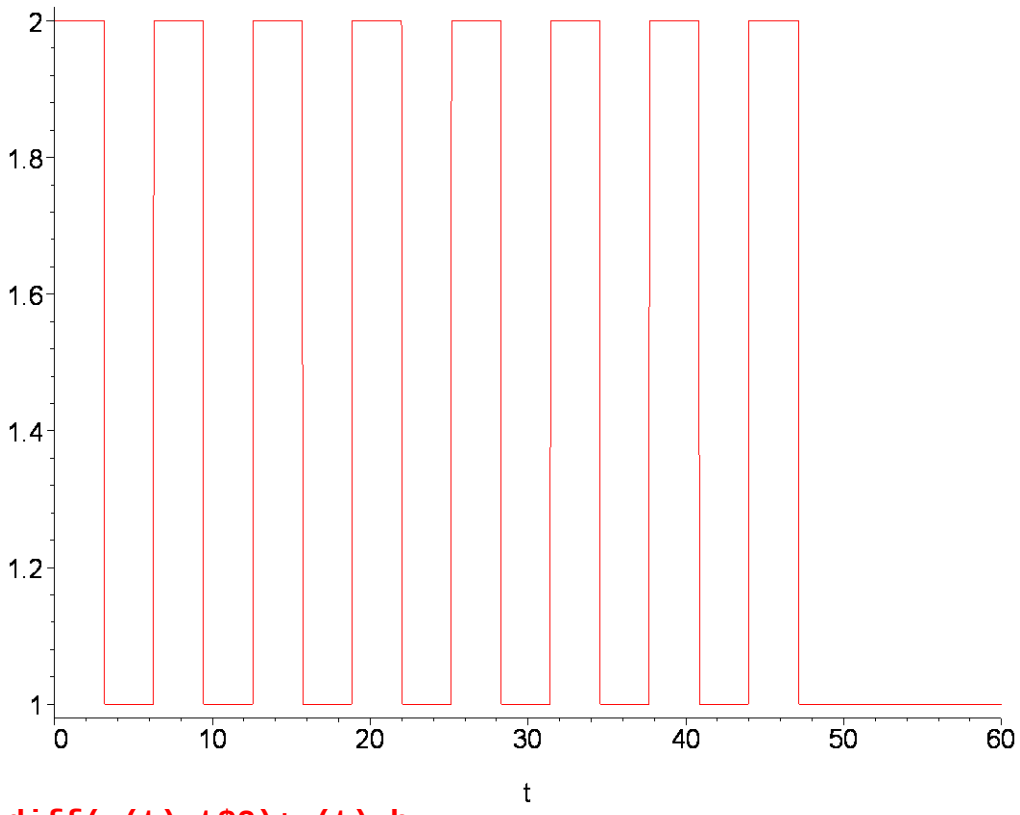


Problem 21

```
> h:=1+sum('((-1)^k)*Heaviside(t-k*Pi)', 'k'=0..15);
```

$$h := 1 + \text{Heaviside}(t) - \text{Heaviside}(t-\pi) + \text{Heaviside}(t-2\pi) - \text{Heaviside}(t-3\pi) + \text{Heaviside}(t-4\pi) - \text{Heaviside}(t-5\pi) + \text{Heaviside}(t-6\pi) - \text{Heaviside}(t-7\pi) + \text{Heaviside}(t-8\pi) - \text{Heaviside}(t-9\pi) + \text{Heaviside}(t-10\pi) - \text{Heaviside}(t-11\pi) + \text{Heaviside}(t-12\pi) - \text{Heaviside}(t-13\pi) + \text{Heaviside}(t-14\pi) - \text{Heaviside}(t-15\pi)$$

```
> plot(h,t=0..60);
```



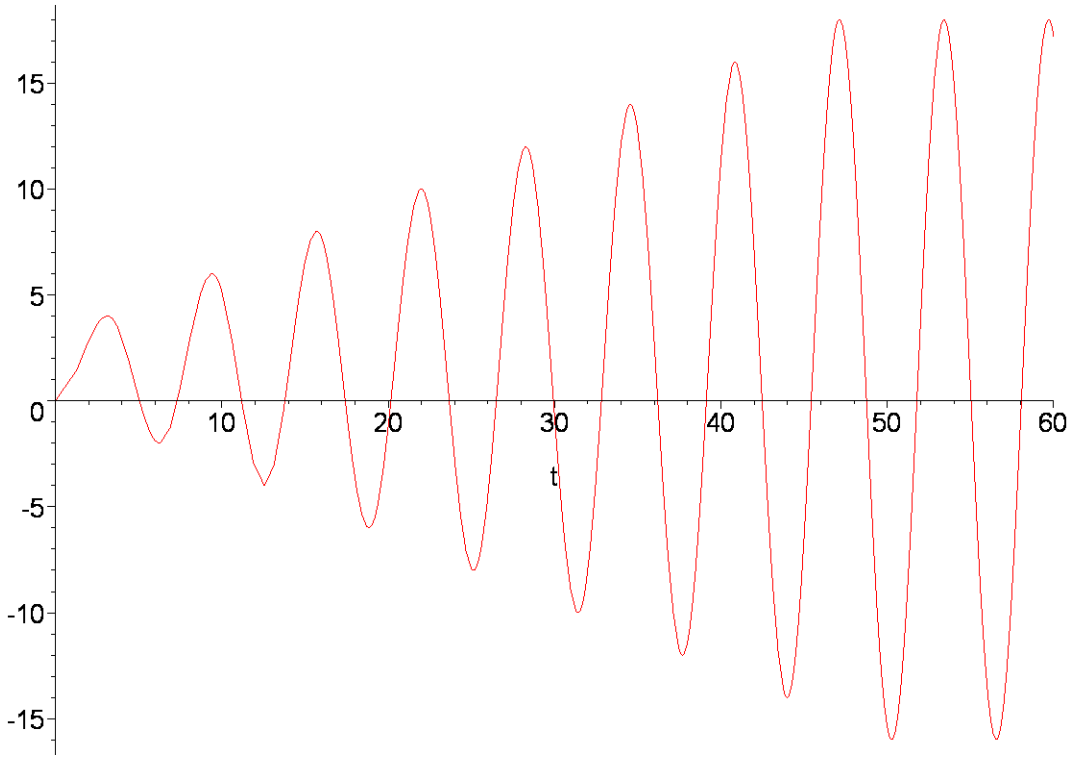

```
> DE21:=diff(y(t),t$2)+y(t)=h;
```

$$DE21 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + y(t) = 1 + \text{Heaviside}(t) - \text{Heaviside}(t-\pi) + \text{Heaviside}(t-2\pi) - \text{Heaviside}(t-3\pi) + \text{Heaviside}(t-4\pi) - \text{Heaviside}(t-5\pi) + \text{Heaviside}(t-6\pi) - \text{Heaviside}(t-7\pi) + \text{Heaviside}(t-8\pi) - \text{Heaviside}(t-9\pi) + \text{Heaviside}(t-10\pi) - \text{Heaviside}(t-11\pi) + \text{Heaviside}(t-12\pi) - \text{Heaviside}(t-13\pi) + \text{Heaviside}(t-14\pi) - \text{Heaviside}(t-15\pi)$$

```
> Y:=dsolve({DE21,y(0)=0,D(y)(0)=0},y(t),method=laplace);
```

$$Y := y(t) = 2\,(-\text{Heaviside}(t-13\pi) - \text{Heaviside}(t-\pi) - \text{Heaviside}(t-15\pi) - \text{Heaviside}(t-3\pi) - \text{Heaviside}(t-5\pi) - \text{Heaviside}(t-7\pi) - \text{Heaviside}(t-9\pi) - \text{Heaviside}(t-11\pi))\cos\left(\frac{1}{2}t\right)^2 + 2\,(\text{Heaviside}(t-4\pi) + \text{Heaviside}(t-6\pi) + \text{Heaviside}(t-8\pi) + \text{Heaviside}(t-10\pi) + \text{Heaviside}(t-12\pi) + \text{Heaviside}(t-14\pi) + \text{Heaviside}(t-2\pi))\sin\left(\frac{1}{2}t\right)^2 - 2\cos(t) + 2$$

```
> plot(rhs(Y),t=0..60);
```



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