Maple Worksheet for Section 6.5, The Dirac Delta Function. For the first problem, we show that detail that Maple can provide (only for those that have had the Calculus Lab). Otherwise, the graphs are shown so that you can get a feeling for how it all works.

```
> restart;
> with(inttrans):  #Defines the integral transforms (e.g., laplace)
> DE01:=diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=Dirac(t-Pi);
```

$$DE01 := \frac{d^2}{dt^2} y(t) + 2 \left(\frac{d}{dt} y(t) \right) + 2\, y(t) = \mathrm{Dirac}(t - \pi) \quad (1)$$

```
> Y1:=laplace(DE01,t,s);
```

$$Y1 := s^2\, laplace(y(t), t, s) - \mathrm{D}(y)(0) - s\, y(0) + 2\, s\, laplace(y(t), t, s) - 2\, y(0) + 2\, laplace(y(t), t, s) = \mathrm{e}^{-s\pi} \quad (2)$$

```
> Y2:=solve(Y1,laplace(y(t),t,s));
```

$$Y2 := \frac{\mathrm{D}(y)(0) + s\, y(0) + 2\, y(0) + \mathrm{e}^{-s\pi}}{s^2 + 2\, s + 2} \quad (3)$$

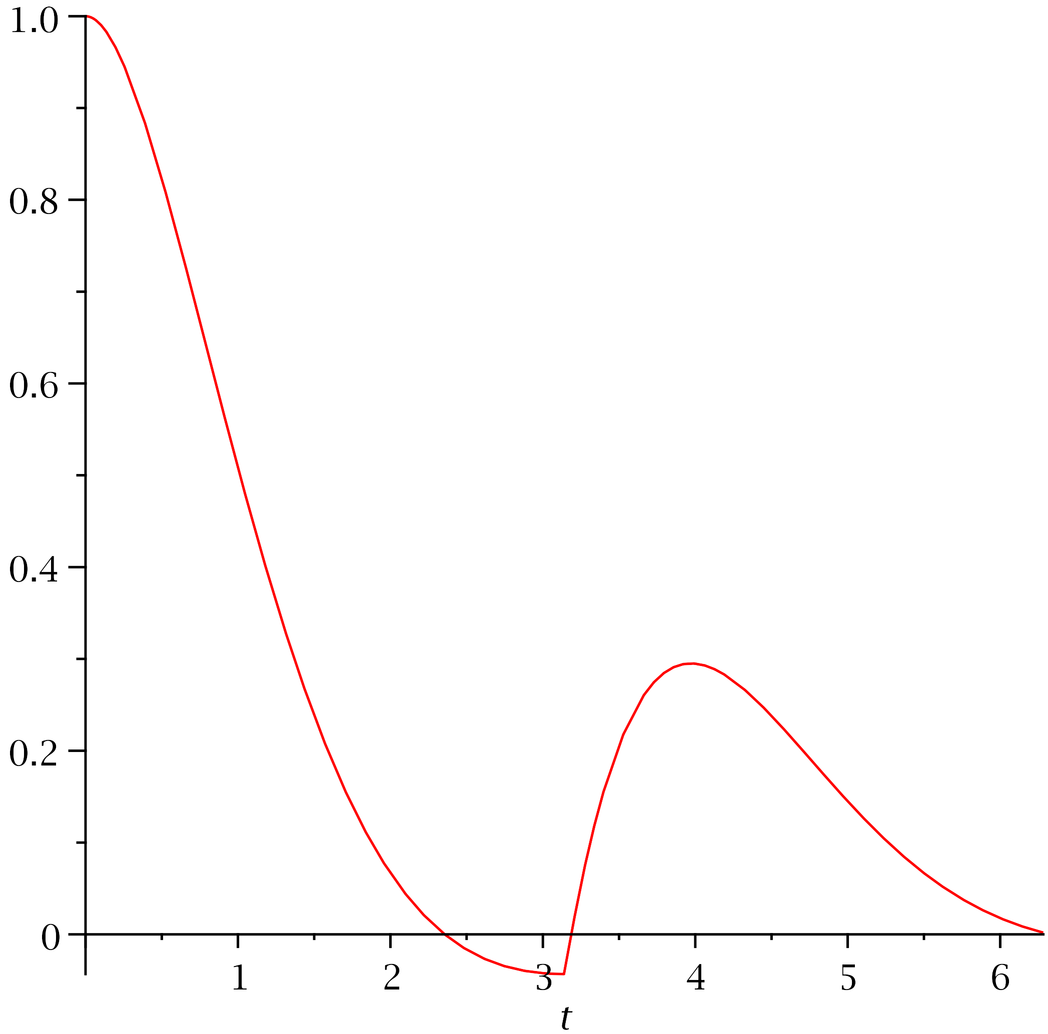
```
> Y3:=subs({y(0)=1,D(y)(0)=0},Y2);
```

$$Y3 := \frac{2 + s + \mathrm{e}^{-s\pi}}{s^2 + 2\, s + 2} \quad (4)$$

```
> Y4:=invlaplace(Y3,s,t);
```

$$Y4 := \mathrm{e}^{-t} (\sin(t) + \cos(t)) - \mathrm{Heaviside}(t - \pi)\, \mathrm{e}^{-t+\pi} \sin(t) \quad (5)$$

```
> plot(Y4,t=0..2*Pi);
```



For the second problem , we'll go ahead and take the shortcut solution and get the plot.

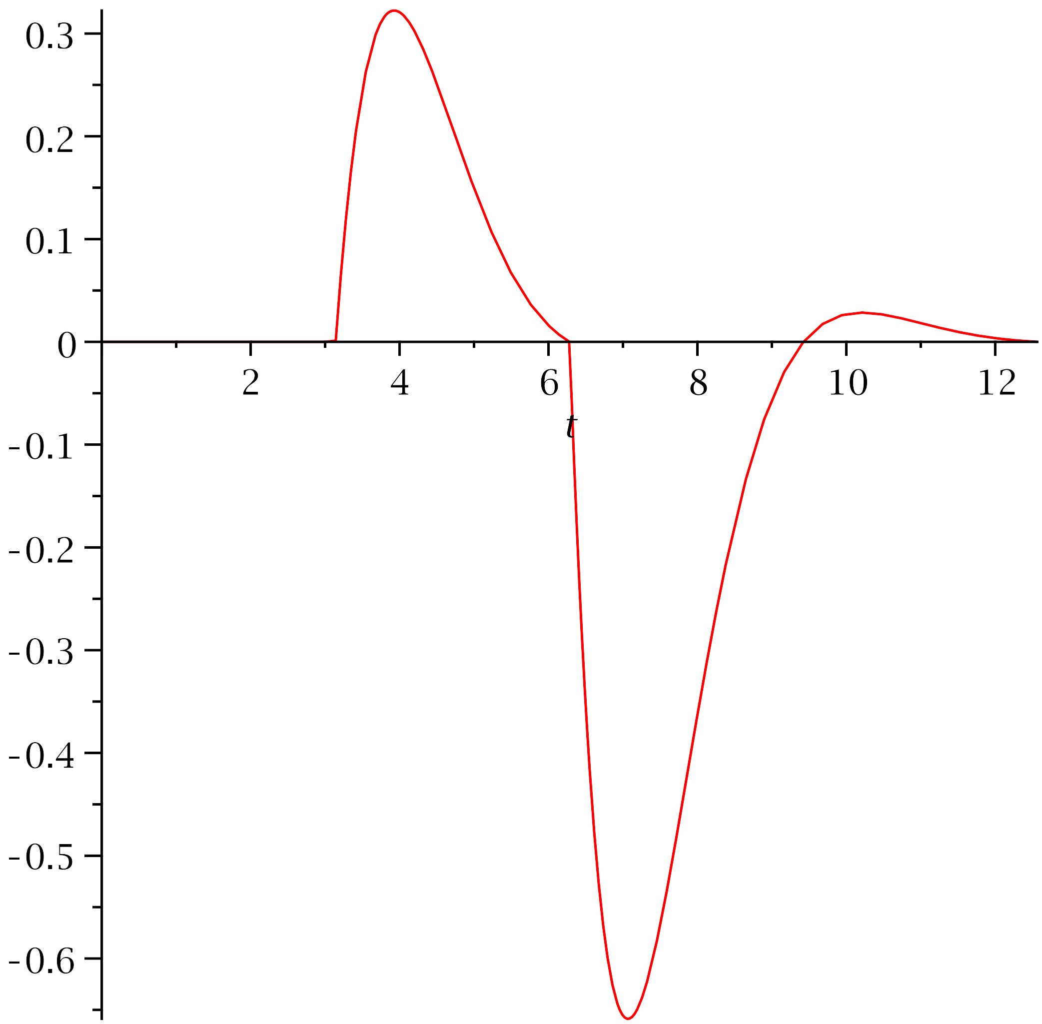
```
> DE02:=diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=Dirac(t-Pi)-Dirac(t-2*Pi);
```

$$DE02 := \frac{d^2}{dt^2} y(t) + 2 \left(\frac{d}{dt} y(t) \right) + 2 y(t) = \mathrm{Dirac}(t-\pi) - \mathrm{Dirac}(t - 2\pi) \quad \textbf{(6)}$$

```
> Y:=dsolve({DE02,y(0)=0,D(y)(0)=0},y(t));
```

$$Y := y(t) = -\sin(t) \left(e^{-t+\pi} \mathrm{Heaviside}(t-\pi) + 2 e^{-t+2\pi} \mathrm{Heaviside}(t - 2\pi) \right) \quad \textbf{(7)}$$

```
> plot(rhs(Y),t=0..4*Pi);
```



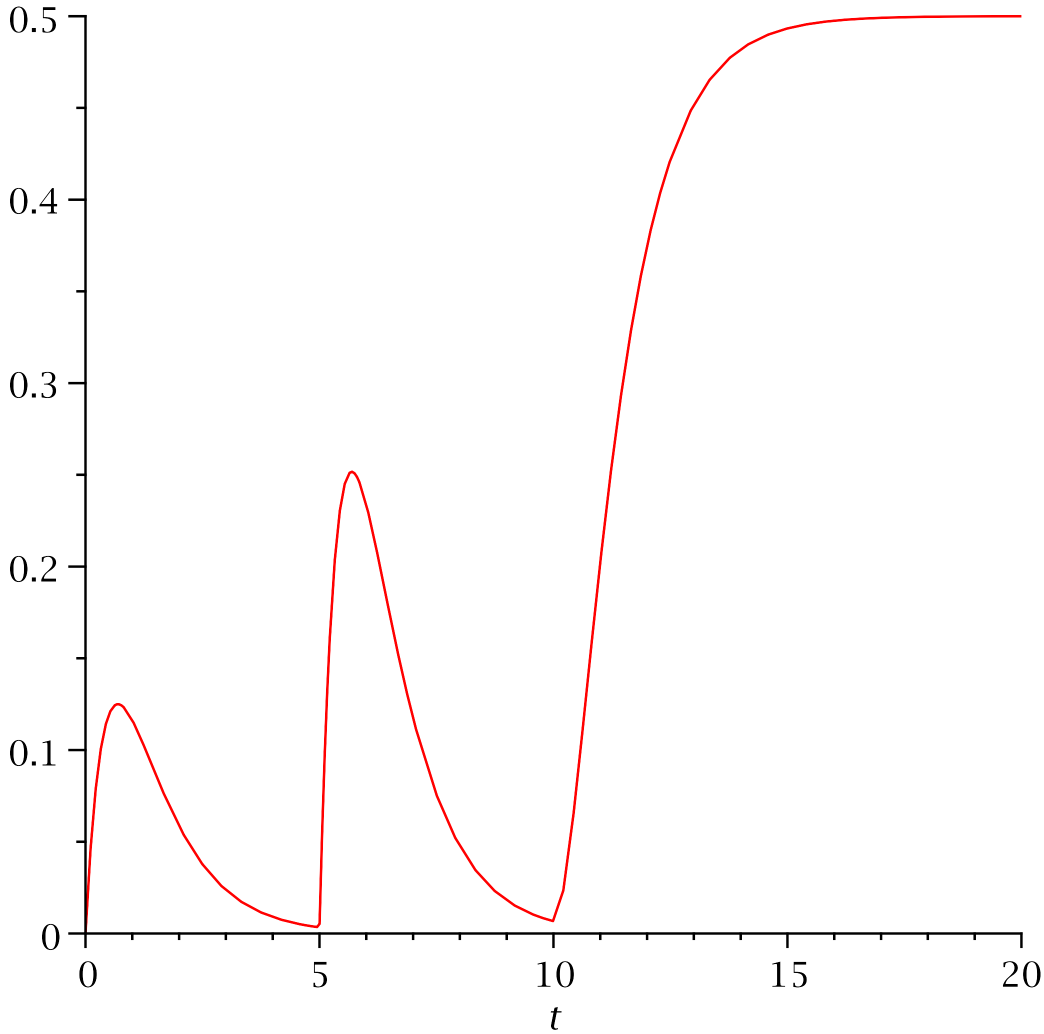

```
> DE03:=diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=Dirac(t-5)+Heaviside
(t-10);
```

$$DE03 := \frac{d^2}{dt^2} y(t) + 3 \left(\frac{d}{dt} y(t)\right) + 2\, y(t) = \mathrm{Dirac}(t-5) + \mathrm{Heaviside}(t-10) \quad \textbf{(8)}$$

```
> Y:=dsolve({DE03,y(0)=0,D(y)(0)=1/2},y(t));
```

$$Y := y(t) = -\mathrm{Heaviside}(t-5)\, e^{10-2t} + \mathrm{Heaviside}(t-5)\, e^{-t+5} + \frac{1}{2} \mathrm{Heaviside}(t-10) - \mathrm{Heaviside}(t-10)\, e^{-t+10} + \frac{1}{2} \mathrm{Heaviside}(t-10)\, e^{20-2t} - \frac{1}{2} e^{-2t} + \frac{1}{2} e^{-t} \quad \textbf{(9)}$$

```
> plot(rhs(Y),t=0..20);
```



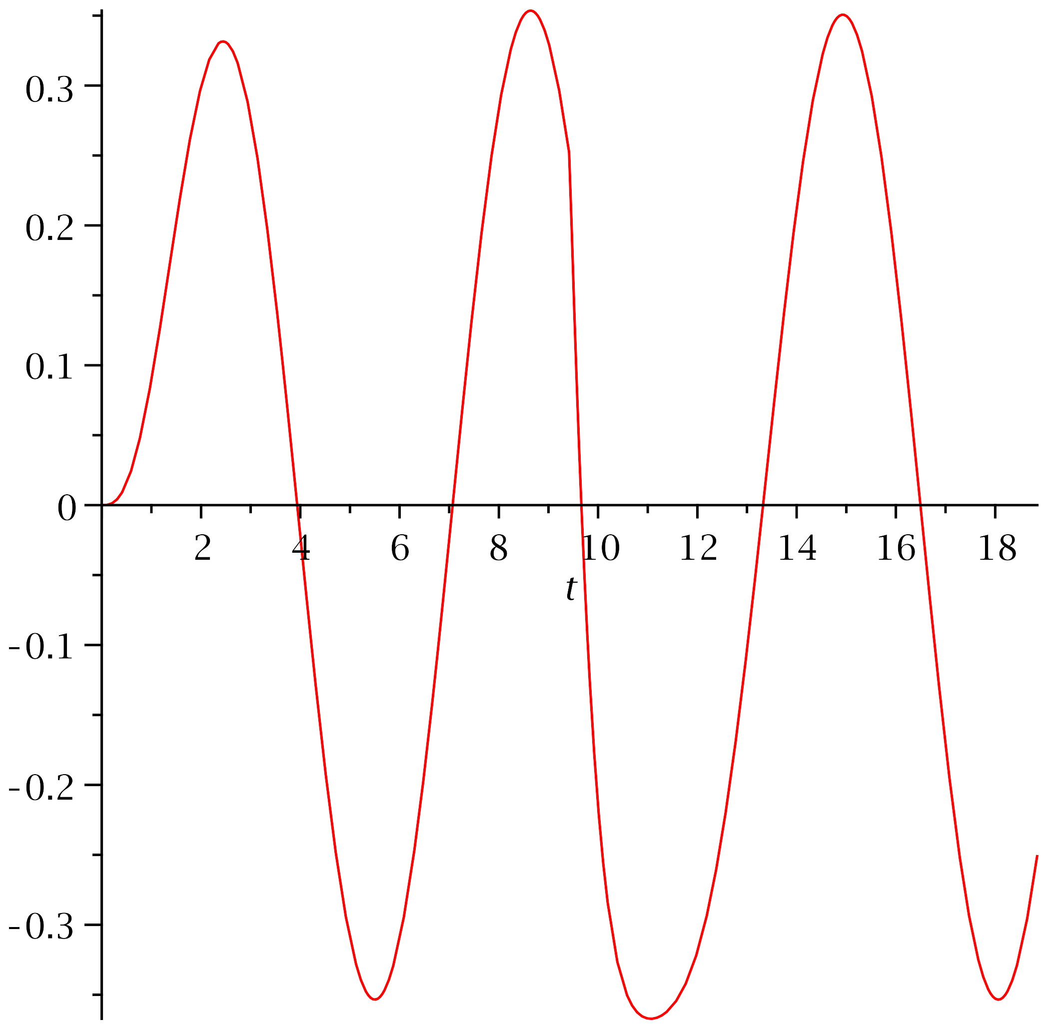

```
> DE05:=diff(y(t),t$2)+2*diff(y(t),t)+3*y(t)=sin(t)-Dirac(t-3*Pi)
;
```

$$DE05 := \frac{d^2}{dt^2} y(t) + 2 \left(\frac{d}{dt} y(t) \right) + 3\, y(t) = \sin(t) - \mathrm{Dirac}(t - 3\,\pi) \quad \textbf{(10)}$$

```
> Y:=dsolve({DE05,y(0)=0,D(y)(0)=0},y(t));
```

$$Y := y(t) = \frac{1}{4}\, e^{-t} \cos\left(\sqrt{2}\; t\right) - \frac{1}{2}\, \sqrt{2}\; \mathrm{Heaviside}(t - 3\,\pi)\, \sin\left(\sqrt{2}\; (t - 3\,\pi)\right) e^{-t + 3\,\pi} + \frac{1}{4}\, \sin(t) - \frac{1}{4}\, \cos(t) \quad \textbf{(11)}$$

```
> plot(rhs(Y),t=0..6*Pi);
```



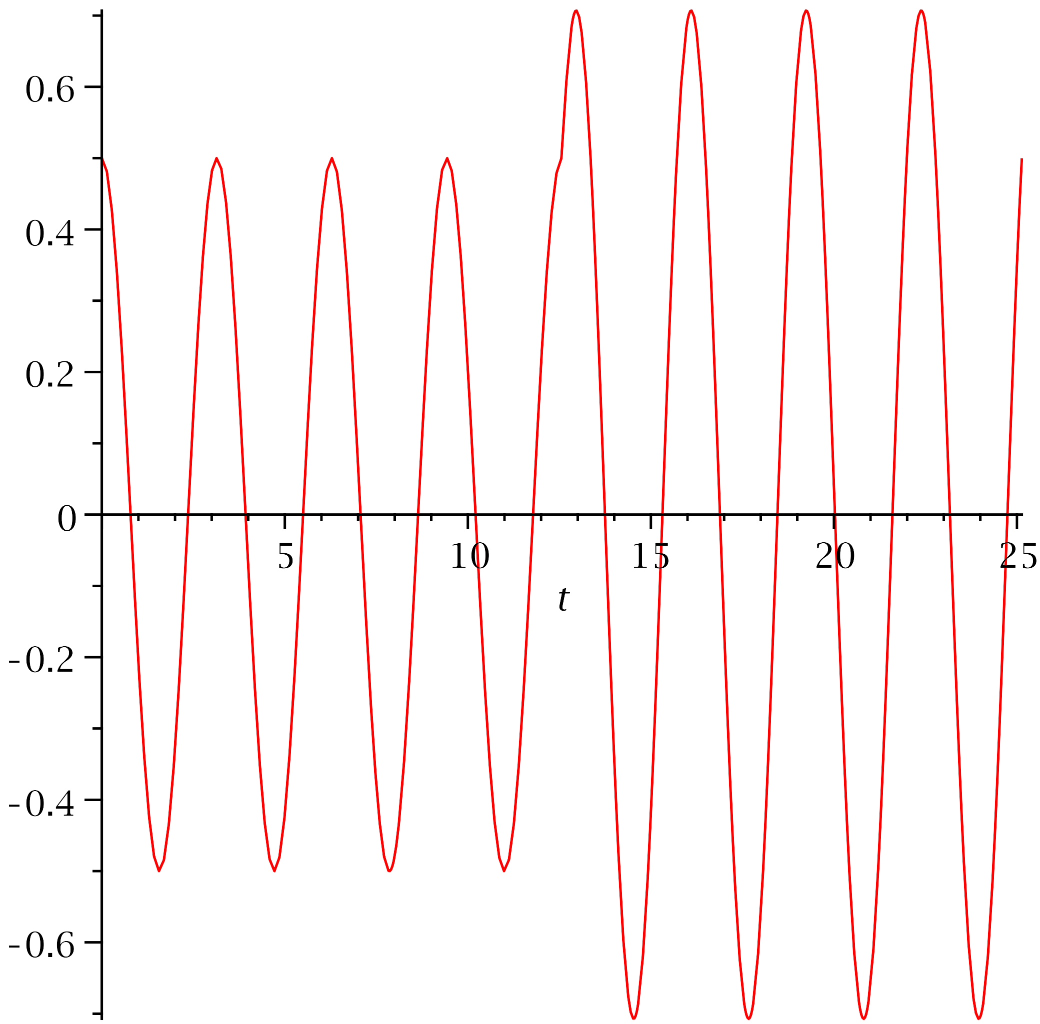

```
> DE06:=diff(y(t),t$2)+4*y(t)=Dirac(t-4*Pi);
```

$$DE06 := \frac{d^2}{dt^2}\, y(t) + 4\, y(t) = \mathrm{Dirac}(t - 4\,\pi) \quad \textbf{(12)}$$

```
> Y:=dsolve({DE06,y(0)=1/2,D(y)(0)=0},y(t));
```

$$Y := y(t) = \frac{1}{2}\cos(2\,t) + \frac{1}{2}\,\mathrm{Heaviside}(t - 4\,\pi)\,\sin(2\,t) \quad \textbf{(13)}$$

```
> plot(rhs(Y),t=0..8*Pi);
```



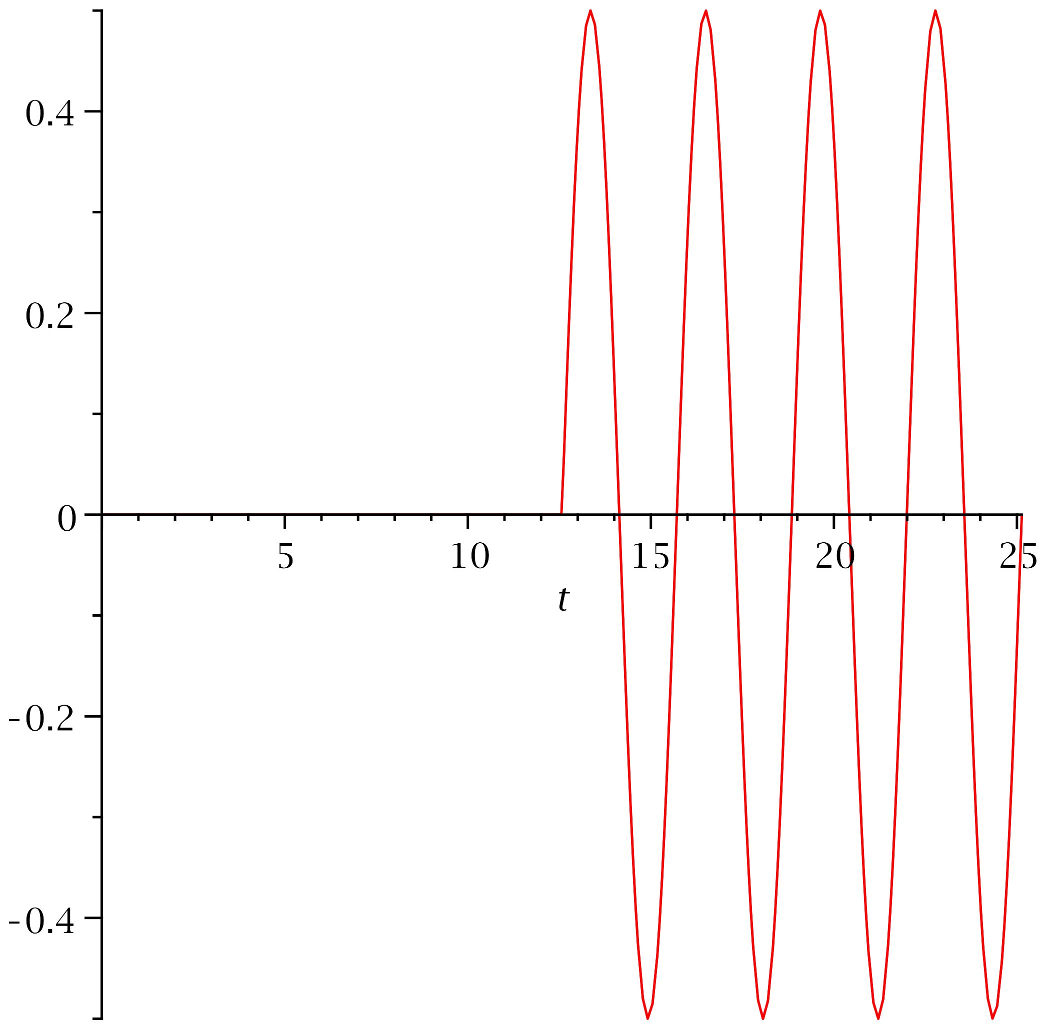

```
> DE08:=diff(y(t),t$2)+4*y(t)=2*Dirac(t-4*Pi);
```

$$DE08 := \frac{d^2}{dt^2}\, y(t) + 4\, y(t) = 2\, \mathrm{Dirac}(t - 4\,\pi) \tag{14}$$

```
> Y:=dsolve({DE06,y(0)=0,D(y)(0)=0},y(t));
```

$$Y := y(t) = \frac{1}{2}\, \mathrm{Heaviside}(t - 4\,\pi)\, \sin(2\, t) \tag{15}$$

```
> plot(rhs(Y),t=0..8*Pi);
```




```
> DE17:=diff(y(t),t$2)+y(t)=sum(Dirac(t-k*Pi),k=1..20);
```

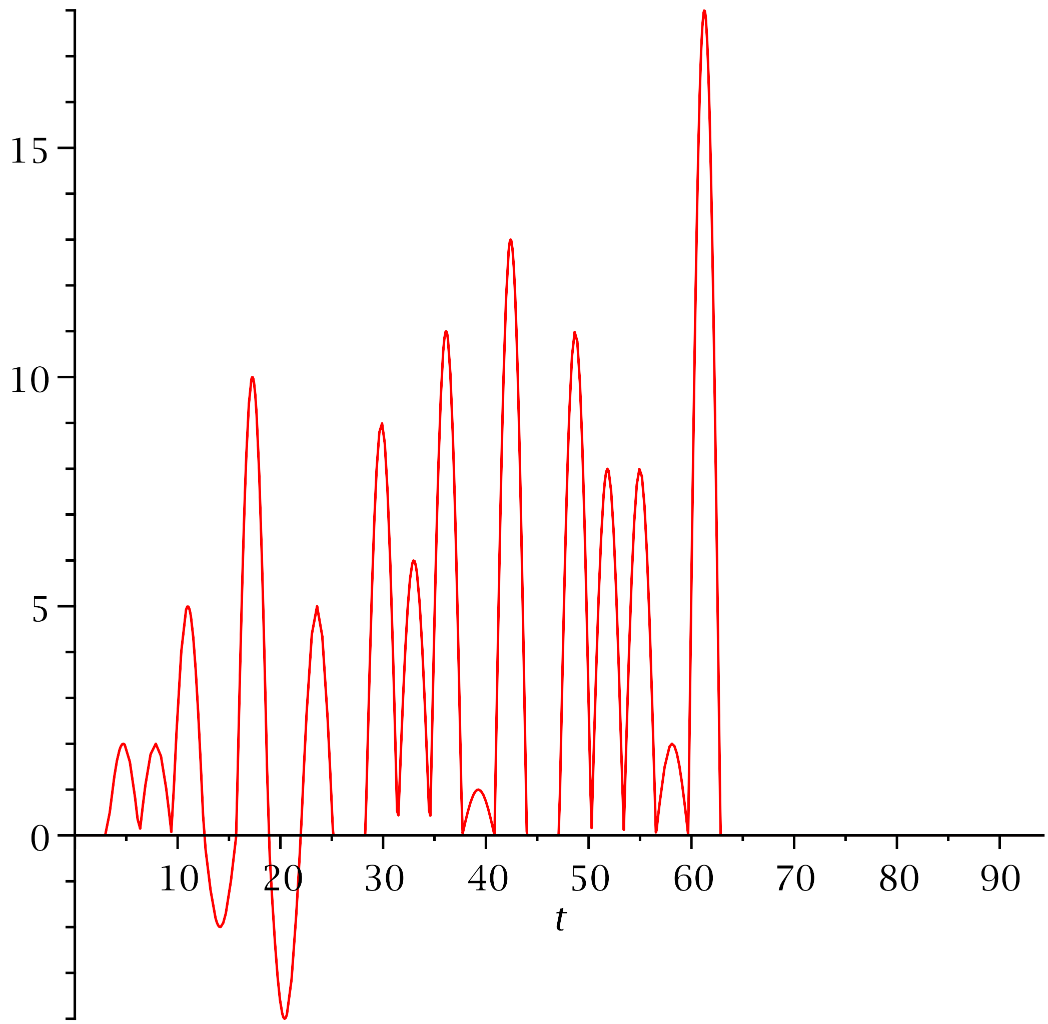
$$DE17 := \frac{d^2}{dt^2}\, y(t) + y(t) = \mathrm{Dirac}(t-\pi) + \mathrm{Dirac}(t-2\pi) + \mathrm{Dirac}(t-3\pi) + \mathrm{Dirac}(t-4\pi) + \mathrm{Dirac}(t-5\pi) + \mathrm{Dirac}(t-6\pi) + \mathrm{Dirac}(t-7\pi) + \mathrm{Dirac}(t-8\pi) + \mathrm{Dirac}(t-9\pi) + \mathrm{Dirac}(t-10\pi) + \mathrm{Dirac}(t-11\pi) + \mathrm{Dirac}(t-12\pi) + \mathrm{Dirac}(t-13\pi) + \mathrm{Dirac}(t-14\pi) + \mathrm{Dirac}(t-15\pi) + \mathrm{Dirac}(t-16\pi) + \mathrm{Dirac}(t-17\pi) + \mathrm{Dirac}(t-18\pi) + \mathrm{Dirac}(t-19\pi) + \mathrm{Dirac}(t-20\pi) \quad \textbf{(16)}$$

```
> Y:=dsolve({DE17,y(0)=0,D(y)(0)=0},y(t));
```

$$Y := y(t) = 3\sin(t)\,\mathrm{Heaviside}(t-4\pi) - 2\sin(t)\,\mathrm{Heaviside}(t-\pi) + 4\sin(t)\,\mathrm{Heaviside}(t-2\pi) + (19\,\mathrm{Heaviside}(t-16\pi) + 12\,\mathrm{Heaviside}(t-12\pi) + 15\,\mathrm{Heaviside}(t-10\pi) - 14\,\mathrm{Heaviside}(t-13\pi) - 17\,\mathrm{Heaviside}(t-11\pi) - 11\,\mathrm{Heaviside}(t-15\pi) + 5\,\mathrm{Heaviside}(t-8\pi) - 7\,\mathrm{Heaviside}(t-3\pi) + 18\,\mathrm{Heaviside}(t-20\pi) - 16\,\mathrm{Heaviside}(t-17\pi) - 20\,\mathrm{Heaviside}(t-19\pi) - \mathrm{Heaviside}(t-7\pi) - 8\,\mathrm{Heaviside}(t-5\pi) + 13\,\mathrm{Heaviside}(t-14\pi) \quad \textbf{(17)}$$

$+ 10\,\mathrm{Heaviside}(t - 18\,\pi) + 6\,\mathrm{Heaviside}(t - 6\,\pi) - 9\,\mathrm{Heaviside}(t - 9\,\pi))$
$\sin(t)$

```
> plot(rhs(Y),t=0..30*Pi);
```



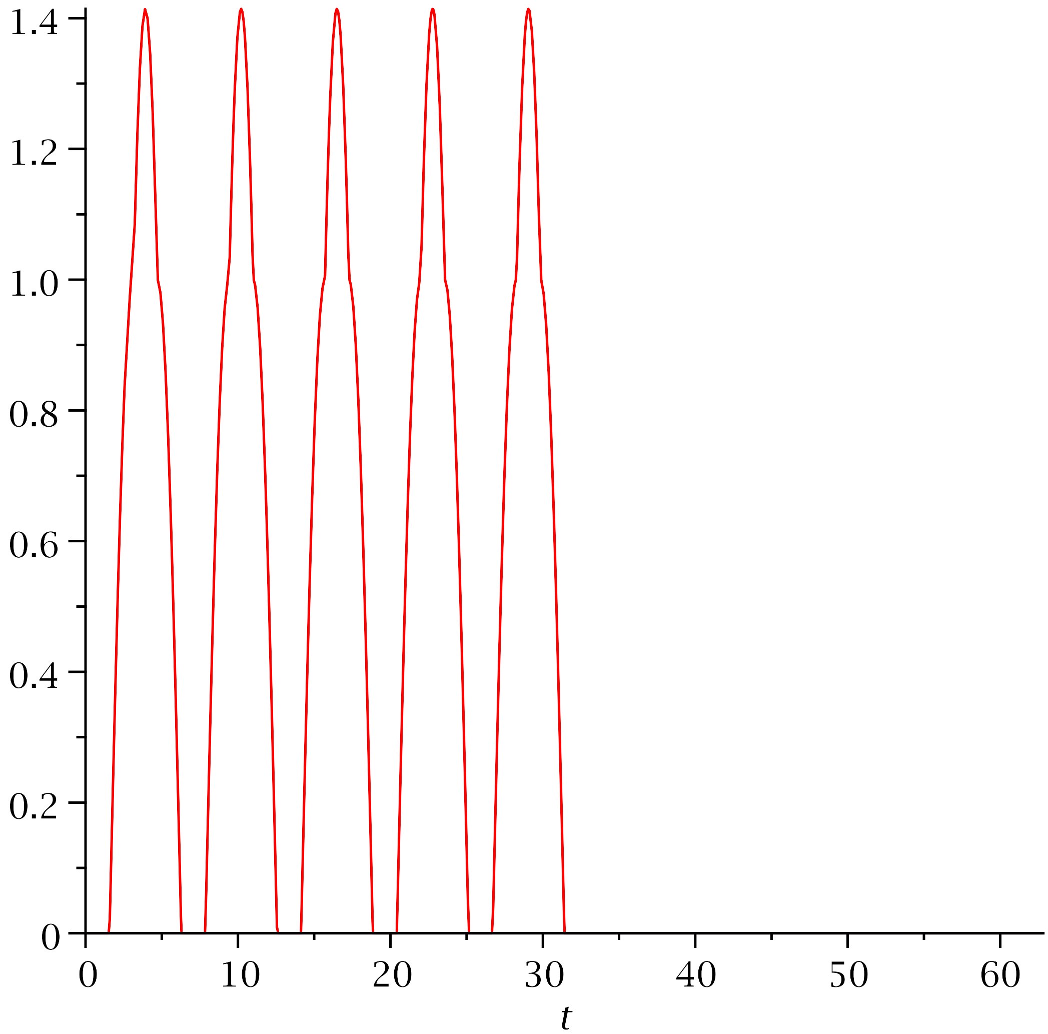

```
> DE19:=diff(y(t),t$2)+y(t)=sum(Dirac(t-k*Pi/2),k=1..20);
```

$$DE19 := \frac{d^2}{dt^2}\,y(t) + y(t) = \mathrm{Dirac}\left(t - \frac{1}{2}\,\pi\right) + \mathrm{Dirac}(t - \pi) + \mathrm{Dirac}\left(t - \frac{3}{2}\,\pi\right) + \mathrm{Dirac}(t - 2\,\pi) + \mathrm{Dirac}\left(t - \frac{5}{2}\,\pi\right) + \mathrm{Dirac}(t - 3\,\pi) + \mathrm{Dirac}\left(t - \frac{7}{2}\,\pi\right) + \mathrm{Dirac}(t - 4\,\pi) + \mathrm{Dirac}\left(t - \frac{9}{2}\,\pi\right) + \mathrm{Dirac}(t - 5\,\pi) + \mathrm{Dirac}\left(t - \frac{11}{2}\,\pi\right) + \mathrm{Dirac}(t - 6\,\pi) + \mathrm{Dirac}\left(t - \frac{13}{2}\,\pi\right) + \mathrm{Dirac}(t - 7\,\pi) + \mathrm{Dirac}\left(t - \frac{15}{2}\,\pi\right) + \mathrm{Dirac}(t - 8\,\pi) + \mathrm{Dirac}\left(t - \frac{17}{2}\,\pi\right) + \mathrm{Dirac}(t - 9\,\pi) + \mathrm{Dirac}\left(t - \frac{19}{2}\,\pi\right) + \mathrm{Dirac}(t - 10\,\pi) \quad \textbf{(18)}$$

```
> Y:=dsolve({DE19,y(0)=0,D(y)(0)=0},y(t), method=laplace);
```

$$Y := y(t) = \big(-\mathrm{Heaviside}(t-\pi) + \mathrm{Heaviside}(t-10\pi) - \mathrm{Heaviside}(t-9\pi) + \mathrm{Heaviside}(t-8\pi) - \mathrm{Heaviside}(t-7\pi) + \mathrm{Heaviside}(t-6\pi) - \mathrm{Heaviside}(t-5\pi) + \mathrm{Heaviside}(t-4\pi) - \mathrm{Heaviside}(t-3\pi) + \mathrm{Heaviside}(t-2\pi)\big)\sin(t) + \left(-\mathrm{Heaviside}\left(t-\frac{1}{2}\pi\right) + \mathrm{Heaviside}\left(t-\frac{19}{2}\pi\right) - \mathrm{Heaviside}\left(t-\frac{17}{2}\pi\right) + \mathrm{Heaviside}\left(t-\frac{15}{2}\pi\right) - \mathrm{Heaviside}\left(t-\frac{13}{2}\pi\right) + \mathrm{Heaviside}\left(t-\frac{11}{2}\pi\right) - \mathrm{Heaviside}\left(t-\frac{9}{2}\pi\right) + \mathrm{Heaviside}\left(t-\frac{7}{2}\pi\right) - \mathrm{Heaviside}\left(t-\frac{5}{2}\pi\right) + \mathrm{Heaviside}\left(t-\frac{3}{2}\pi\right)\right)\cos(t) \quad \textbf{(19)}$$

```
> plot(rhs(Y),t=0..20*Pi);
```




```
> restart;
>
```