Maple Worksheet for Section 6.5, The Dirac Delta Function. For the first problem, we show that detail that Maple can provide (only for those that have had the Calculus Lab). Otherwise, the graphs are shown so that you can get a feeling for how it all works.

```
> restart;
> with(inttrans):  #Defines the integral transforms (e.g., laplace)
> DE01:=diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=Dirac(t-Pi);
```

$$DE01 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 2\left(\frac{\partial}{\partial t} y(t)\right) + 2\, y(t) = \mathrm{Dirac}(t-\pi)$$

```
> Y1:=laplace(DE01,t,s);
```

$$Y1 := s\,(s\,\mathrm{laplace}(y(t),t,s) - y(0)) - D(y)(0) + 2\,s\,\mathrm{laplace}(y(t),t,s) - 2\,y(0) + 2\,\mathrm{laplace}(y(t),t,s) = e^{(-s\,\pi)}$$

```
> Y2:=solve(Y1,laplace(y(t),t,s));
```

$$Y2 := \frac{s\,y(0) + D(y)(0) + 2\,y(0) + e^{(-s\,\pi)}}{s^2 + 2\,s + 2}$$

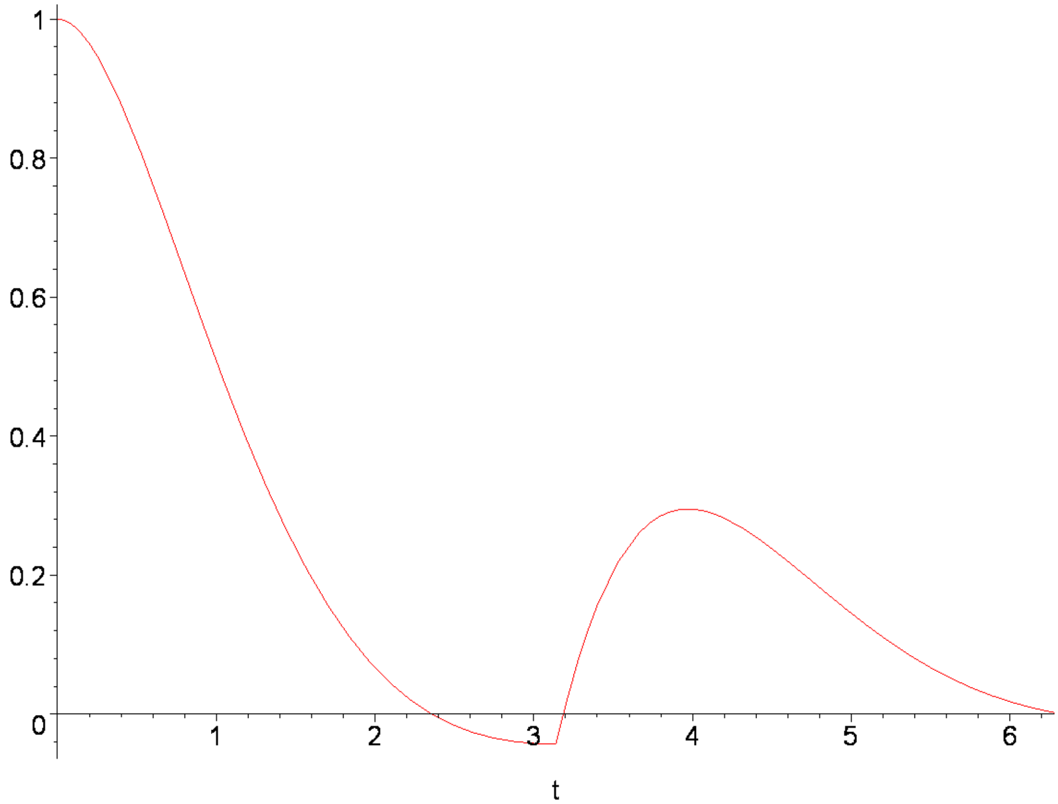
```
> Y3:=subs({y(0)=1,D(y)(0)=0},Y2);
```

$$Y3 := \frac{s + 2 + e^{(-s\,\pi)}}{s^2 + 2\,s + 2}$$

```
> Y4:=invlaplace(Y3,s,t);
```

$$Y4 := e^{(-t)}\cos(t) + e^{(-t)}\sin(t) - \mathrm{Heaviside}(t-\pi)\,e^{(-t+\pi)}\sin(t)$$

```
> plot(Y4,t=0..2*Pi);
```



For the second problem , we'll go ahead and take the shortcut solution and get the plot.

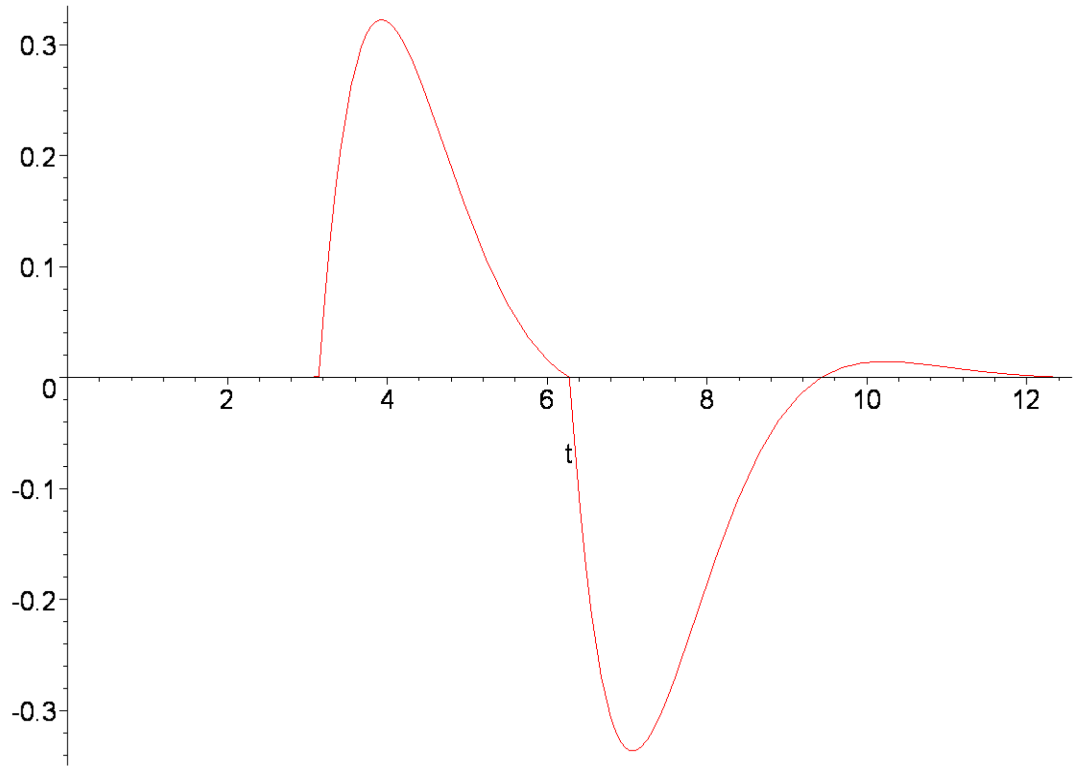
```
> DE02:=diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=Dirac(t-Pi)-Dirac(t-2*Pi);
```

$$DE02 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 2\left(\frac{\partial}{\partial t} y(t)\right) + 2\, y(t) = \mathrm{Dirac}(t-\pi) - \mathrm{Dirac}(t-2\,\pi)$$

```
> Y:=dsolve({DE02,y(0)=0,D(y)(0)=0},y(t));
```

$$Y := y(t) = -\sin(t)\,(e^{(-t+\pi)}\,\mathrm{Heaviside}(t-\pi) + e^{(-t+2\,\pi)}\,\mathrm{Heaviside}(t-2\,\pi))$$

```
> plot(rhs(Y),t=0..4*Pi);
```



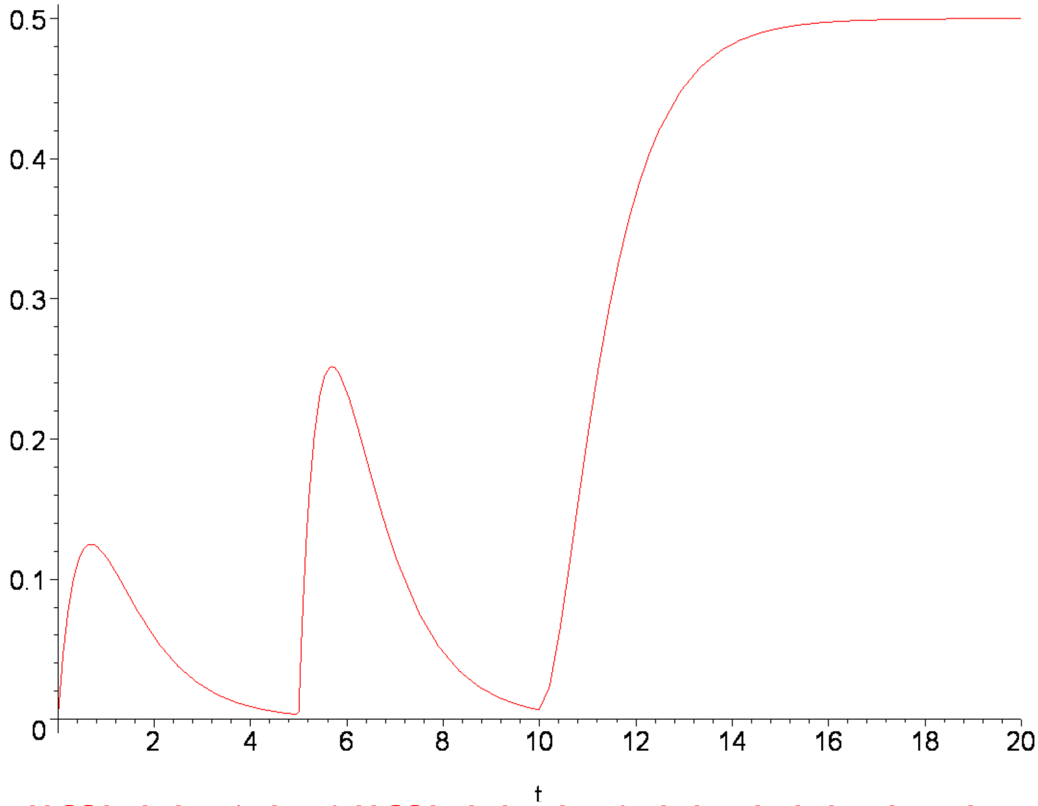
```
> DE03:=diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=Dirac(t-5)+Heaviside(t-10);
```

$$DE03 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 3\left(\frac{\partial}{\partial t} y(t)\right) + 2\, y(t) = \mathrm{Dirac}(t-5) + \mathrm{Heaviside}(t-10)$$

```
> Y:=dsolve({DE03,y(0)=0,D(y)(0)=1/2},y(t));
```

$$Y := y(t) = -\mathrm{Heaviside}(t-5)\, e^{(10-2t)} + \mathrm{Heaviside}(t-5)\, e^{(-t+5)} + \frac{1}{2}\mathrm{Heaviside}(t-10) - \mathrm{Heaviside}(t-10)\, e^{(-t+10)} + \frac{1}{2}\mathrm{Heaviside}(t-10)\, e^{(20-2t)} - \frac{1}{2} e^{(-2t)} + \frac{1}{2} e^{(-t)}$$

```
> plot(rhs(Y),t=0..20);
```



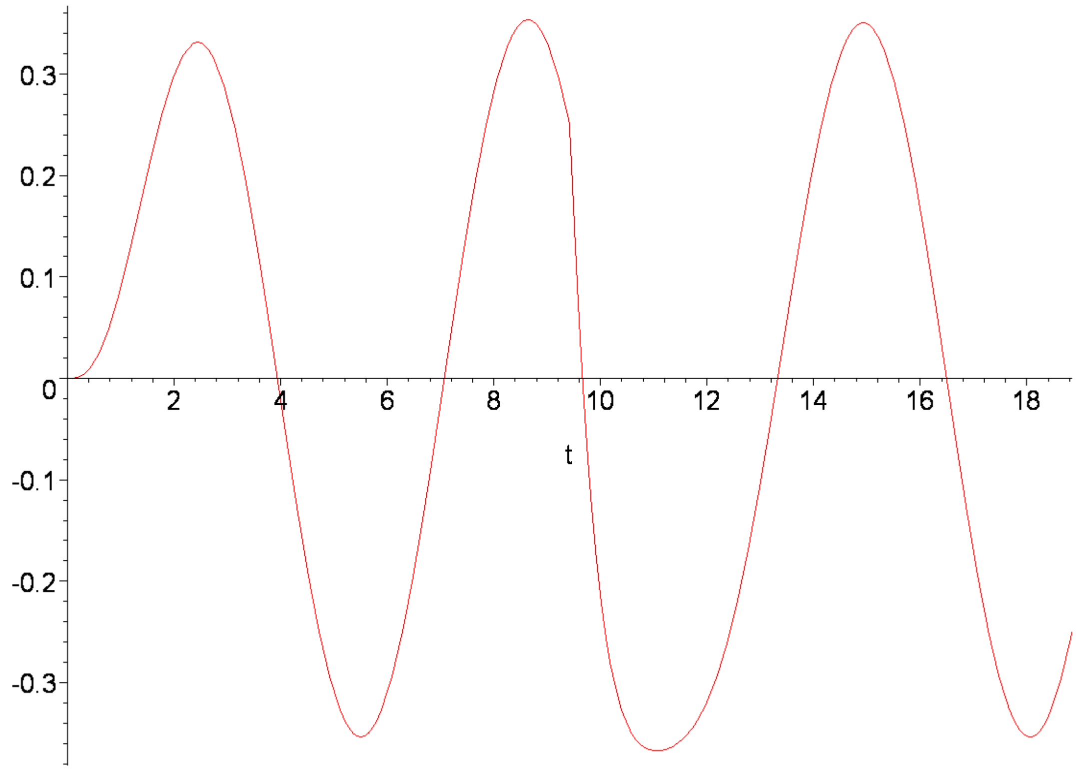

```
> DE05:=diff(y(t),t$2)+2*diff(y(t),t)+3*y(t)=sin(t)-Dirac(t-3*Pi);
```

$$DE05 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 2\left(\frac{\partial}{\partial t} y(t)\right) + 3\, y(t) = \sin(t) - \mathrm{Dirac}(t - 3\,\pi)$$

```
> Y:=dsolve({DE05,y(0)=0,D(y)(0)=0},y(t));
```

$$Y := y(t) = \frac{1}{4} e^{(-t)} \cos(\sqrt{2}\, t) - \frac{1}{4}\cos(t) + \frac{1}{2}\sqrt{2}\, \mathrm{Heaviside}(t - 3\,\pi)\sin(-\sqrt{2}\, t + 3\sqrt{2}\,\pi)\, e^{(-t + 3\,\pi)} + \frac{1}{4}\sin(t)$$

```
> plot(rhs(Y),t=0..6*Pi);
```



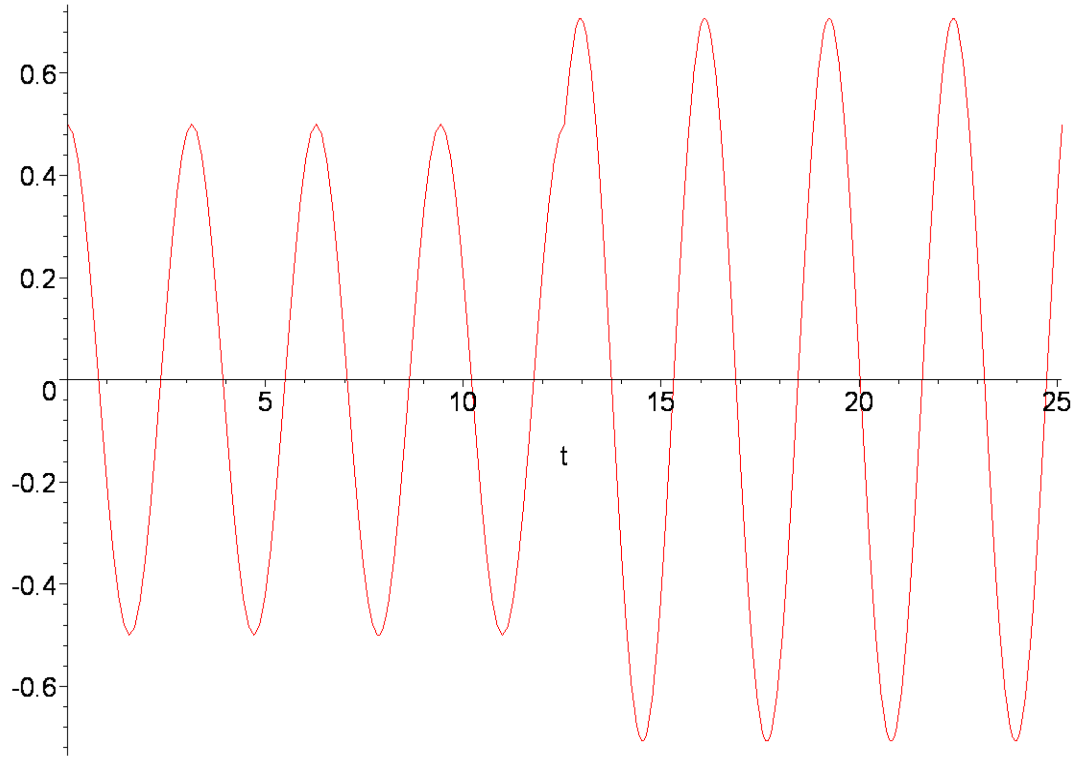

```
> DE06:=diff(y(t),t$2)+4*y(t)=Dirac(t-4*Pi);
```

$$DE06 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 4\, y(t) = \mathrm{Dirac}(t - 4\,\pi)$$

```
> Y:=dsolve({DE06,y(0)=1/2,D(y)(0)=0},y(t),method=laplace);
```

$$Y := y(t) = \frac{1}{2}\cos(2\,t) + \frac{1}{2}\,\mathrm{Heaviside}(t - 4\,\pi)\,\sin(2\,t - 8\,\pi)$$

```
> plot(rhs(Y),t=0..8*Pi);
```



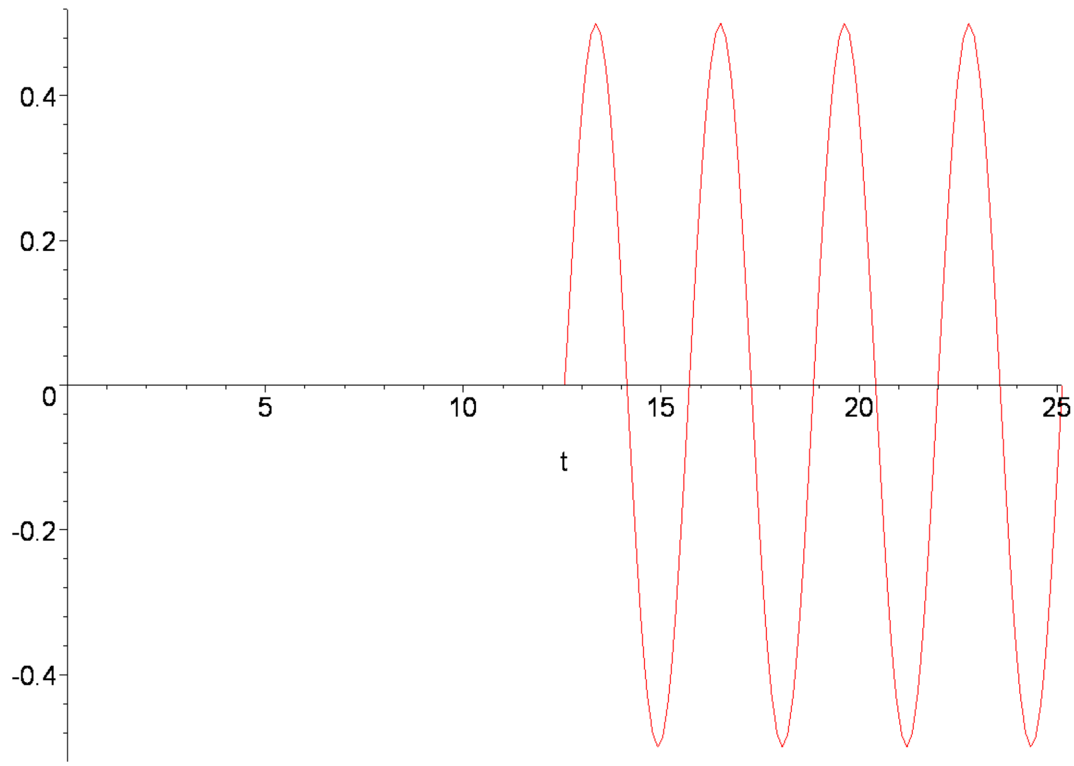

```
> DE08:=diff(y(t),t$2)+4*y(t)=2*Dirac(t-4*Pi);
```

$$DE08 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 4\, y(t) = 2\, \mathrm{Dirac}(t - 4\,\pi)$$

```
> Y:=dsolve({DE06,y(0)=0,D(y)(0)=0},y(t),method=laplace);
```

$$Y := y(t) = \frac{1}{2} \mathrm{Heaviside}(t - 4\,\pi) \sin(2\,t - 8\,\pi)$$

```
> plot(rhs(Y),t=0..8*Pi);
```



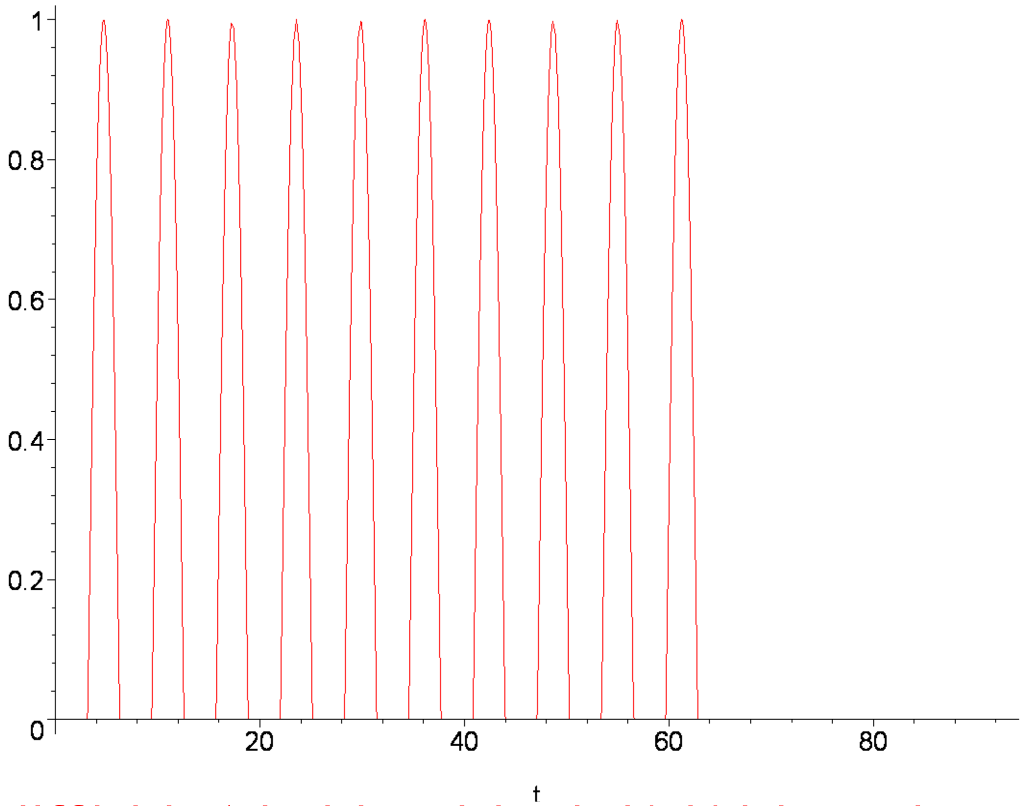

```
> DE17:=diff(y(t),t$2)+y(t)=sum(Dirac(t-k*Pi),k=1..20);
```

$$DE17 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + y(t) = \mathrm{Dirac}(t-\pi) + \mathrm{Dirac}(t-2\pi) + \mathrm{Dirac}(t-3\pi) + \mathrm{Dirac}(t-4\pi) + \mathrm{Dirac}(t-5\pi) + \mathrm{Dirac}(t-6\pi) + \mathrm{Dirac}(t-7\pi) + \mathrm{Dirac}(t-8\pi) + \mathrm{Dirac}(t-9\pi) + \mathrm{Dirac}(t-10\pi) + \mathrm{Dirac}(t-11\pi) + \mathrm{Dirac}(t-12\pi) + \mathrm{Dirac}(t-13\pi) + \mathrm{Dirac}(t-14\pi) + \mathrm{Dirac}(t-15\pi) + \mathrm{Dirac}(t-16\pi) + \mathrm{Dirac}(t-17\pi) + \mathrm{Dirac}(t-18\pi) + \mathrm{Dirac}(t-19\pi) + \mathrm{Dirac}(t-20\pi)$$

```
> Y:=dsolve({DE17,y(0)=0,D(y)(0)=0},y(t));
```

$$Y := y(t) = -(\mathrm{Heaviside}(t-\pi) - \mathrm{Heaviside}(t-2\pi) + \mathrm{Heaviside}(t-3\pi) - \mathrm{Heaviside}(t-4\pi) + \mathrm{Heaviside}(t-5\pi) - \mathrm{Heaviside}(t-6\pi) + \mathrm{Heaviside}(t-7\pi) - \mathrm{Heaviside}(t-8\pi) + \mathrm{Heaviside}(t-9\pi) - \mathrm{Heaviside}(t-10\pi) + \mathrm{Heaviside}(t-11\pi) - \mathrm{Heaviside}(t-12\pi) + \mathrm{Heaviside}(t-13\pi) - \mathrm{Heaviside}(t-14\pi) + \mathrm{Heaviside}(t-15\pi) - \mathrm{Heaviside}(t-16\pi) + \mathrm{Heaviside}(t-17\pi) - \mathrm{Heaviside}(t-18\pi) + \mathrm{Heaviside}(t-19\pi) - \mathrm{Heaviside}(t-20\pi))\sin(t)$$

```
> plot(rhs(Y),t=0..30*Pi);
```



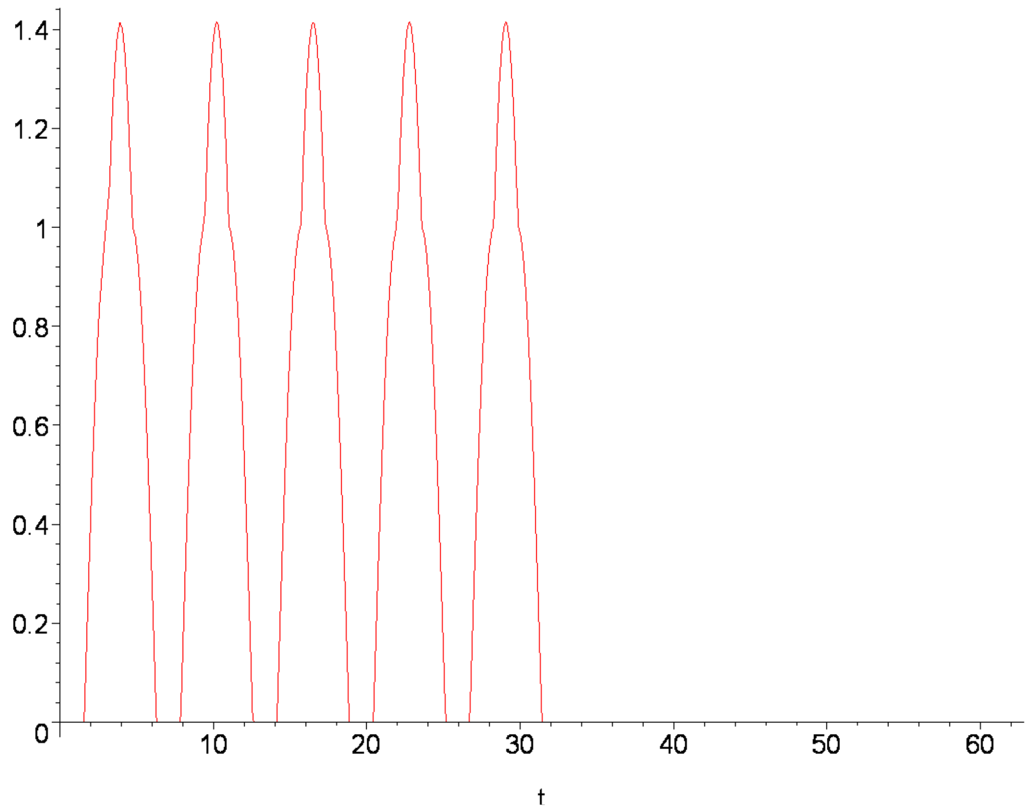

```
> DE19:=diff(y(t),t$2)+y(t)=sum(Dirac(t-k*Pi/2),k=1..20);
```

$$DE19 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + y(t) = \mathrm{Dirac}\left(t - \frac{1}{2}\pi\right) + \mathrm{Dirac}(t - \pi) + \mathrm{Dirac}\left(t - \frac{3}{2}\pi\right) + \mathrm{Dirac}(t - 2\pi)$$
$$+ \mathrm{Dirac}\left(t - \frac{5}{2}\pi\right) + \mathrm{Dirac}(t - 3\pi) + \mathrm{Dirac}\left(t - \frac{7}{2}\pi\right) + \mathrm{Dirac}(t - 4\pi) + \mathrm{Dirac}\left(t - \frac{9}{2}\pi\right)$$
$$+ \mathrm{Dirac}(t - 5\pi) + \mathrm{Dirac}\left(t - \frac{11}{2}\pi\right) + \mathrm{Dirac}(t - 6\pi) + \mathrm{Dirac}\left(t - \frac{13}{2}\pi\right) + \mathrm{Dirac}(t - 7\pi)$$
$$+ \mathrm{Dirac}\left(t - \frac{15}{2}\pi\right) + \mathrm{Dirac}(t - 8\pi) + \mathrm{Dirac}\left(t - \frac{17}{2}\pi\right) + \mathrm{Dirac}(t - 9\pi) + \mathrm{Dirac}\left(t - \frac{19}{2}\pi\right)$$
$$+ \mathrm{Dirac}(t - 10\pi)$$

```
> Y:=dsolve({DE19,y(0)=0,D(y)(0)=0},y(t), method=laplace);
```

$$Y := y(t) = \left(-\mathrm{Heaviside}\left(t - \frac{1}{2}\pi\right) + \mathrm{Heaviside}\left(t - \frac{15}{2}\pi\right) + \mathrm{Heaviside}\left(t - \frac{3}{2}\pi\right)\right.$$
$$- \mathrm{Heaviside}\left(t - \frac{17}{2}\pi\right) - \mathrm{Heaviside}\left(t - \frac{5}{2}\pi\right) + \mathrm{Heaviside}\left(t - \frac{19}{2}\pi\right) + \mathrm{Heaviside}\left(t - \frac{7}{2}\pi\right)$$
$$\left.- \mathrm{Heaviside}\left(t - \frac{9}{2}\pi\right) + \mathrm{Heaviside}\left(t - \frac{11}{2}\pi\right) - \mathrm{Heaviside}\left(t - \frac{13}{2}\pi\right)\right) \cos(t) + (\mathrm{Heaviside}(t - 4\pi)$$
$$- \mathrm{Heaviside}(t - 5\pi) + \mathrm{Heaviside}(t - 6\pi) - \mathrm{Heaviside}(t - 7\pi) - \mathrm{Heaviside}(t - \pi)$$
$$+ \mathrm{Heaviside}(t - 8\pi) + \mathrm{Heaviside}(t - 2\pi) - \mathrm{Heaviside}(t - 9\pi) - \mathrm{Heaviside}(t - 3\pi)$$
$$+ \mathrm{Heaviside}(t - 10\pi)) \sin(t)$$

```
> plot(rhs(Y),t=0..20*Pi);
```




```
> restart;
>
```