Maple Worksheet: Problems 15, 16, 23, 24, Section 5.2

```
> DE15:=diff(y(x),x$2)-x*diff(y(x),x)-y(x)=0;
```

$$DE15 := \frac{d^2}{dx^2} y(x) - x\left(\frac{d}{dx} y(x)\right) - y(x) = 0 \quad (1)$$

```
> Order:=5;
```

$$Order := 5 \quad (2)$$

```
> Y1:=dsolve({DE15,y(0)=2,D(y)(0)=1},y(x),'series');
```

$$Y1 := y(x) = 2 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + O(x^5) \quad (3)$$

```
> Yp15_4:=convert(rhs(Y1),polynom);
```

$$Yp15\_4 := 2 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 \quad (4)$$

```
> Order:=6;
```

$$Order := 6 \quad (5)$$

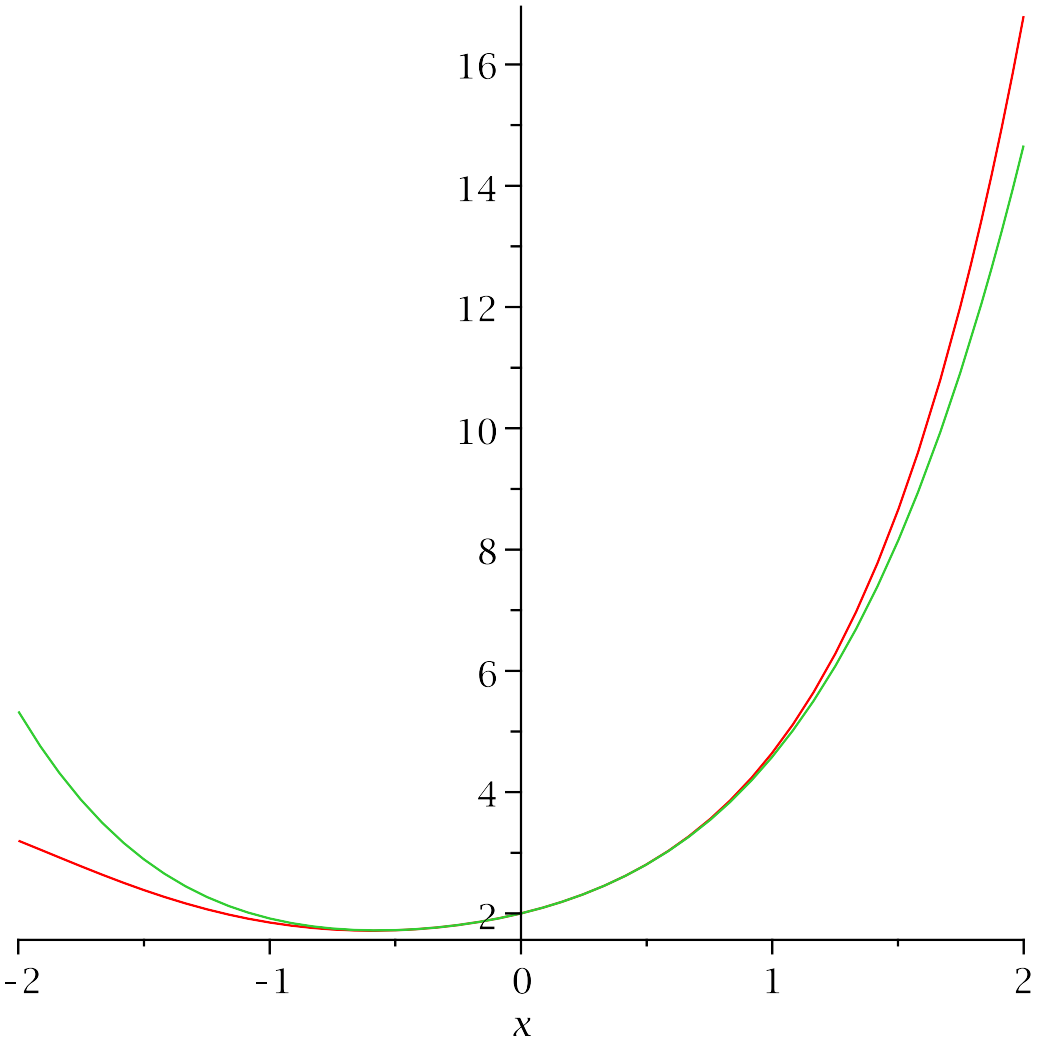
```
> Y2:=dsolve({DE15,y(0)=2,D(y)(0)=1},y(x),'series');
```

$$Y2 := y(x) = 2 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{15}x^5 + O(x^6) \quad (6)$$

```
> Yp15_5:=convert(rhs(Y2),polynom);
```

$$Yp15\_5 := 2 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{15}x^5 \quad (7)$$

```
> plot({Yp15_4,Yp15_5},x=-2..2);
```



Problem 16:

```
> DE16:=(2+x^2)*diff(y(x),x$2)-x*diff(y(x),x)+4*y(x)=0;
```

$$DE16 := (2+x^2)\left(\frac{d^2}{dx^2}\,y(x)\right) - x\left(\frac{d}{dx}\,y(x)\right) + 4\,y(x) = 0 \qquad (8)$$

```
> Order:=5;
```

$$Order := 5 \qquad (9)$$

```
> Y1:=dsolve({DE16,y(0)=-1,D(y)(0)=3},y(x),'series');
```

$$Y1 := y(x) = -1 + 3\,x + x^2 - \frac{3}{4}\,x^3 - \frac{1}{6}\,x^4 + O(x^5) \qquad (10)$$

```
> Yp15_4:=convert(rhs(Y1),polynom);
```

$$Yp15\_4 := -1 + 3\,x + x^2 - \frac{3}{4}\,x^3 - \frac{1}{6}\,x^4 \qquad (11)$$

```
> Order:=6;
```

$$Order := 6 \qquad (12)$$

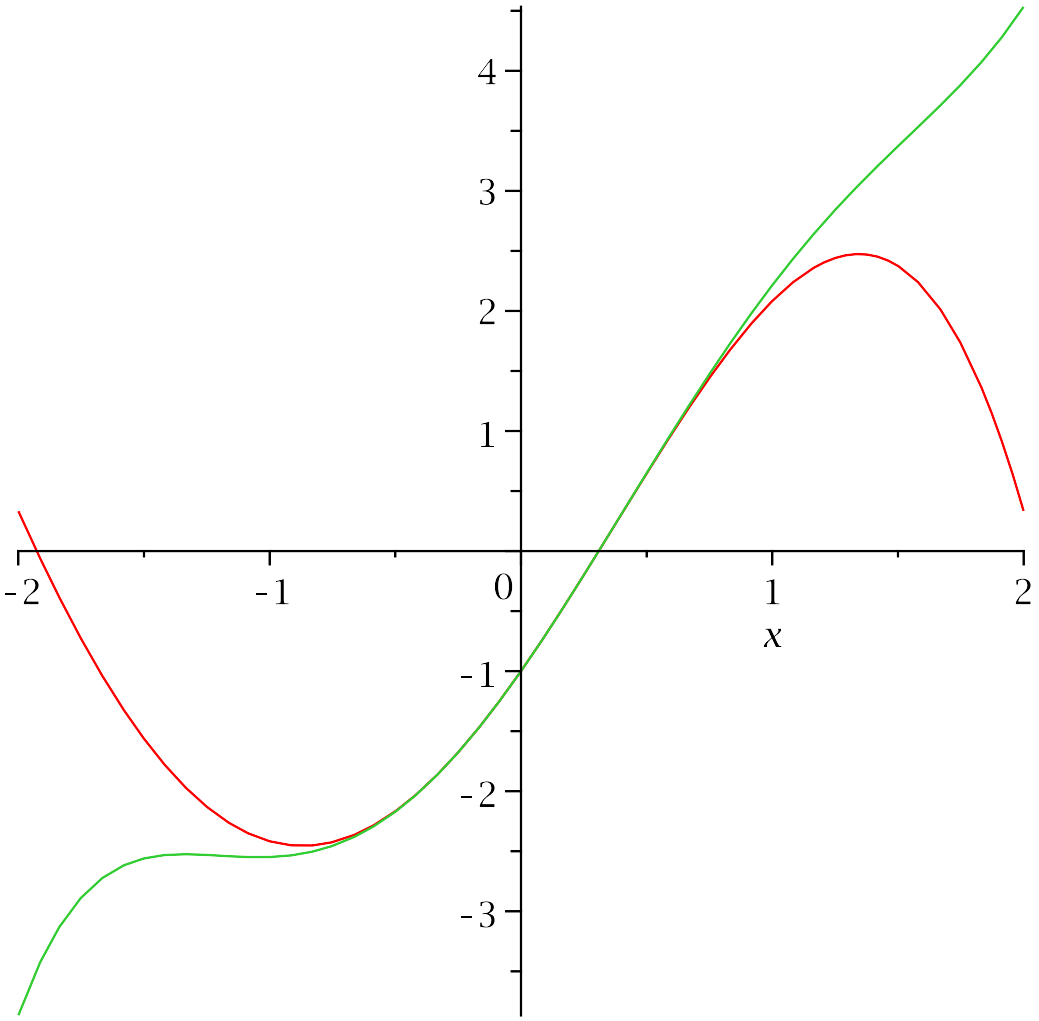
```
> Y2:=dsolve({DE16,y(0)=-1,D(y)(0)=3},y(x),'series');
```

$$Y2 := y(x) = -1 + 3\,x + x^2 - \frac{3}{4}\,x^3 - \frac{1}{6}\,x^4 + \frac{21}{160}\,x^5 + O(x^6) \tag{13}$$

```
> Yp15_5:=convert(rhs(Y2),polynom);
```

$$Yp15\_5 := -1 + 3\,x + x^2 - \frac{3}{4}\,x^3 - \frac{1}{6}\,x^4 + \frac{21}{160}\,x^5 \tag{14}$$

```
> plot({Yp15_4,Yp15_5},x=-2..2);
```



Problems 23 and 24 ask us to expand the interval and look at more terms of the series.

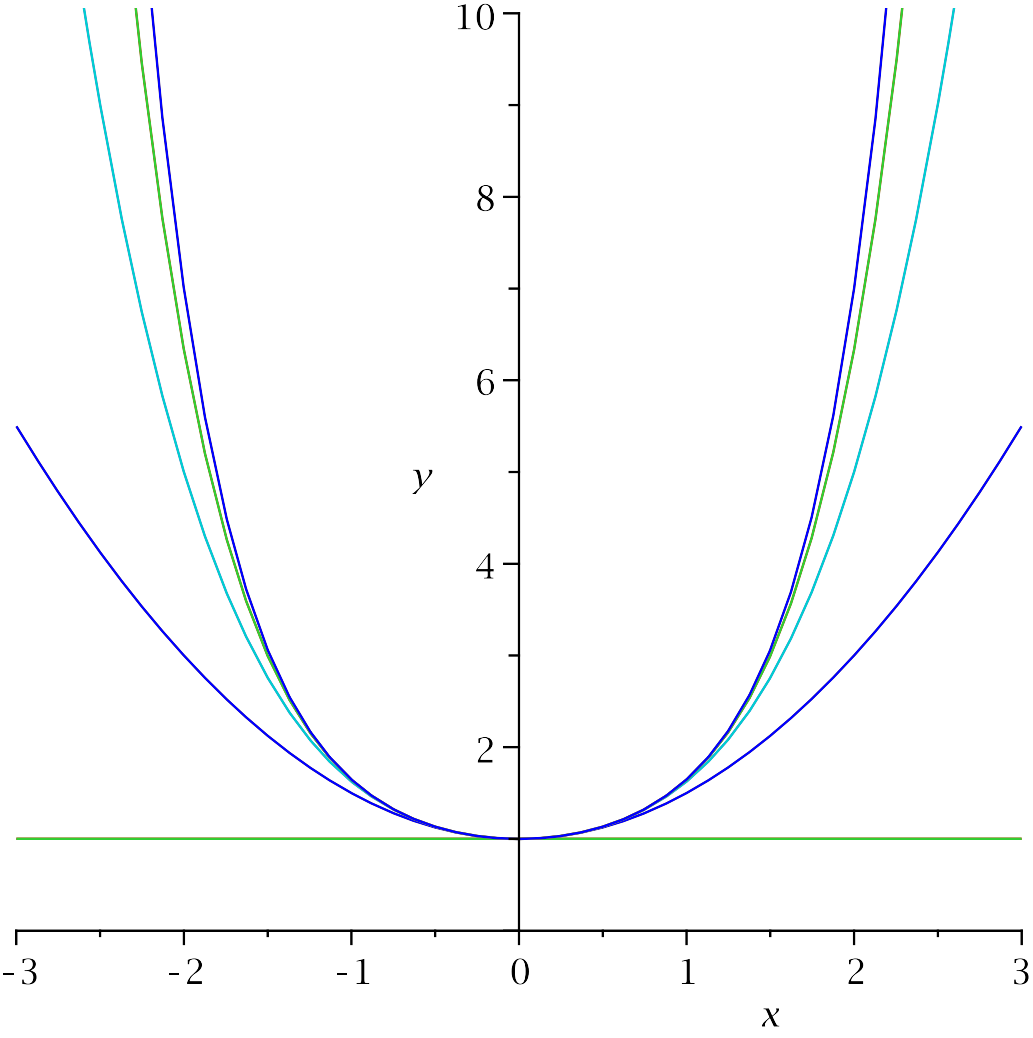
The solution for problem 23 is not quite as nice as the ones in the text:

```
> for j from 1 to 10 do
>     Order:=j:
>     Y:=dsolve({DE15,y(0)=1,D(y)(0)=0},y(x),'series'):
>     Yp[j]:=convert(rhs(Y),polynom):
> end do:
> Yout:=[seq(Yp[j],j=1..10)];
```

(15)

$$Yout := \left[1, 1, 1+\frac{1}{2}x^2, 1+\frac{1}{2}x^2, 1+\frac{1}{2}x^2+\frac{1}{8}x^4, 1+\frac{1}{2}x^2+\frac{1}{8}x^4, 1+\frac{1}{2}x^2 + \frac{1}{8}x^4+\frac{1}{48}x^6, 1+\frac{1}{2}x^2+\frac{1}{8}x^4+\frac{1}{48}x^6, 1+\frac{1}{2}x^2+\frac{1}{8}x^4+\frac{1}{48}x^6 + \frac{1}{384}x^8, 1+\frac{1}{2}x^2+\frac{1}{8}x^4+\frac{1}{48}x^6+\frac{1}{384}x^8\right] \quad (15)$$

```
> plot(Yout,x=-3..3,y=0..10);
```



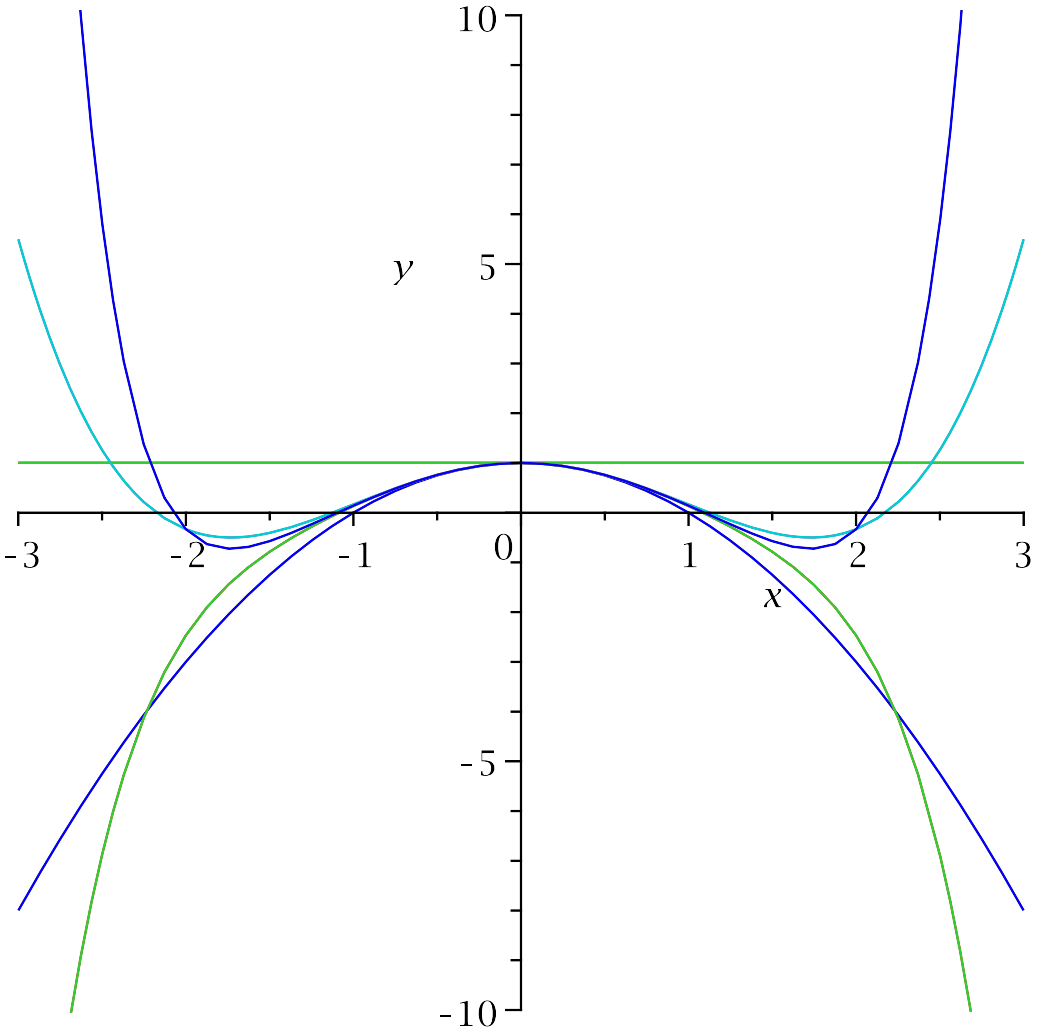
Same for Problem 24:

```
> for j from 1 to 20 do
>     Order:=j:
>     Y:=dsolve({DE16,y(0)=1,D(y)(0)=0},y(x),'series'):
>     Yp[j]:=convert(rhs(Y),polynom):
> end do:
> Yout:=[seq(Yp[j],j=1..10)];
```

(16)

$$Yout := \left[1,\ 1,\ 1-x^2,\ 1-x^2,\ 1-x^2+\frac{1}{6}x^4,\ 1-x^2+\frac{1}{6}x^4,\ 1-x^2+\frac{1}{6}x^4-\frac{1}{30}x^6,\ 1-x^2+\frac{1}{6}x^4-\frac{1}{30}x^6+\frac{1}{120}x^8,\ 1-x^2+\frac{1}{6}x^4-\frac{1}{30}x^6+\frac{1}{120}x^8\right] \quad (16)$$

```
> plot(Yout,x=-3..3,y=-10..10);
```



Just for fun, let's look at the example from the text and reproduce the figure:

```
> DE:=diff(y(x),x$2)-x*y(x)=0;
```

$$DE := \frac{d^2}{dx^2}y(x) - x\,y(x) = 0 \quad (17)$$

```
> for j from 1 to 16 do
>     Order:=3*j+1:
>     Y:=dsolve({DE,y(0)=1,D(y)(0)=0},y(x),'series'):
```

```
>     Yp[j]:=convert(rhs(Y),polynom):
> end do:
> Yout:=[seq(Yp[j],j=1..10)];
```

$$Yout := \left[1+\frac{1}{6}x^3,\ 1+\frac{1}{6}x^3+\frac{1}{180}x^6,\ 1+\frac{1}{6}x^3+\frac{1}{180}x^6+\frac{1}{12960}x^9,\ 1+\frac{1}{6}x^3\right. \tag{18}$$

$$+\frac{1}{180}x^6+\frac{1}{12960}x^9+\frac{1}{1710720}x^{12},\ 1+\frac{1}{6}x^3+\frac{1}{180}x^6+\frac{1}{12960}x^9$$

$$+\frac{1}{1710720}x^{12}+\frac{1}{359251200}x^{15},\ 1+\frac{1}{6}x^3+\frac{1}{180}x^6+\frac{1}{12960}x^9$$

$$+\frac{1}{1710720}x^{12}+\frac{1}{359251200}x^{15}+\frac{1}{109930867200}x^{18},\ 1+\frac{1}{6}x^3+\frac{1}{180}x^6$$

$$+\frac{1}{12960}x^9+\frac{1}{1710720}x^{12}+\frac{1}{359251200}x^{15}+\frac{1}{109930867200}x^{18}$$

$$+\frac{1}{46170964224000}x^{21},\ 1+\frac{1}{6}x^3+\frac{1}{180}x^6+\frac{1}{12960}x^9+\frac{1}{1710720}x^{12}$$

$$+\frac{1}{359251200}x^{15}+\frac{1}{109930867200}x^{18}+\frac{1}{46170964224000}x^{21}$$

$$+\frac{1}{25486372251648000}x^{24},\ 1+\frac{1}{6}x^3+\frac{1}{180}x^6+\frac{1}{12960}x^9+\frac{1}{1710720}x^{12}$$

$$+\frac{1}{359251200}x^{15}+\frac{1}{109930867200}x^{18}+\frac{1}{46170964224000}x^{21}$$

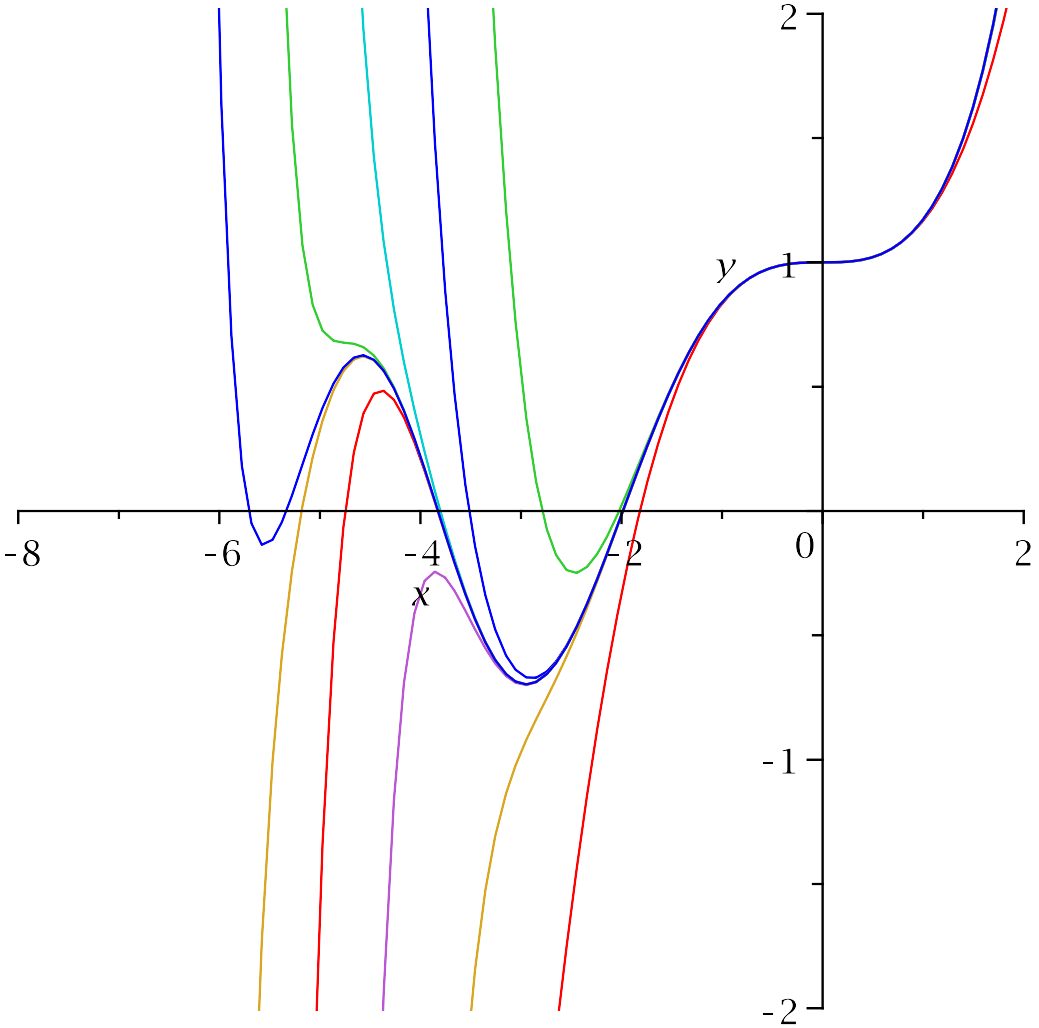
$$+\frac{1}{25486372251648000}x^{24}+\frac{1}{17891433320656896000}x^{27},\ 1+\frac{1}{6}x^3$$

$$+\frac{1}{180}x^6+\frac{1}{12960}x^9+\frac{1}{1710720}x^{12}+\frac{1}{359251200}x^{15}$$

$$+\frac{1}{109930867200}x^{18}+\frac{1}{46170964224000}x^{21}+\frac{1}{25486372251648000}x^{24}$$

$$\left.+\frac{1}{17891433320656896000}x^{27}+\frac{1}{1556554698897149952000}x^{30}\right]$$

```
> plot(Yout,x=-8..2,y=-2..2,numpoints=100);
```


2
y
1
-8
-6
-4
-2
0
2
x
-1
-2

>
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