From Beating to Resonance

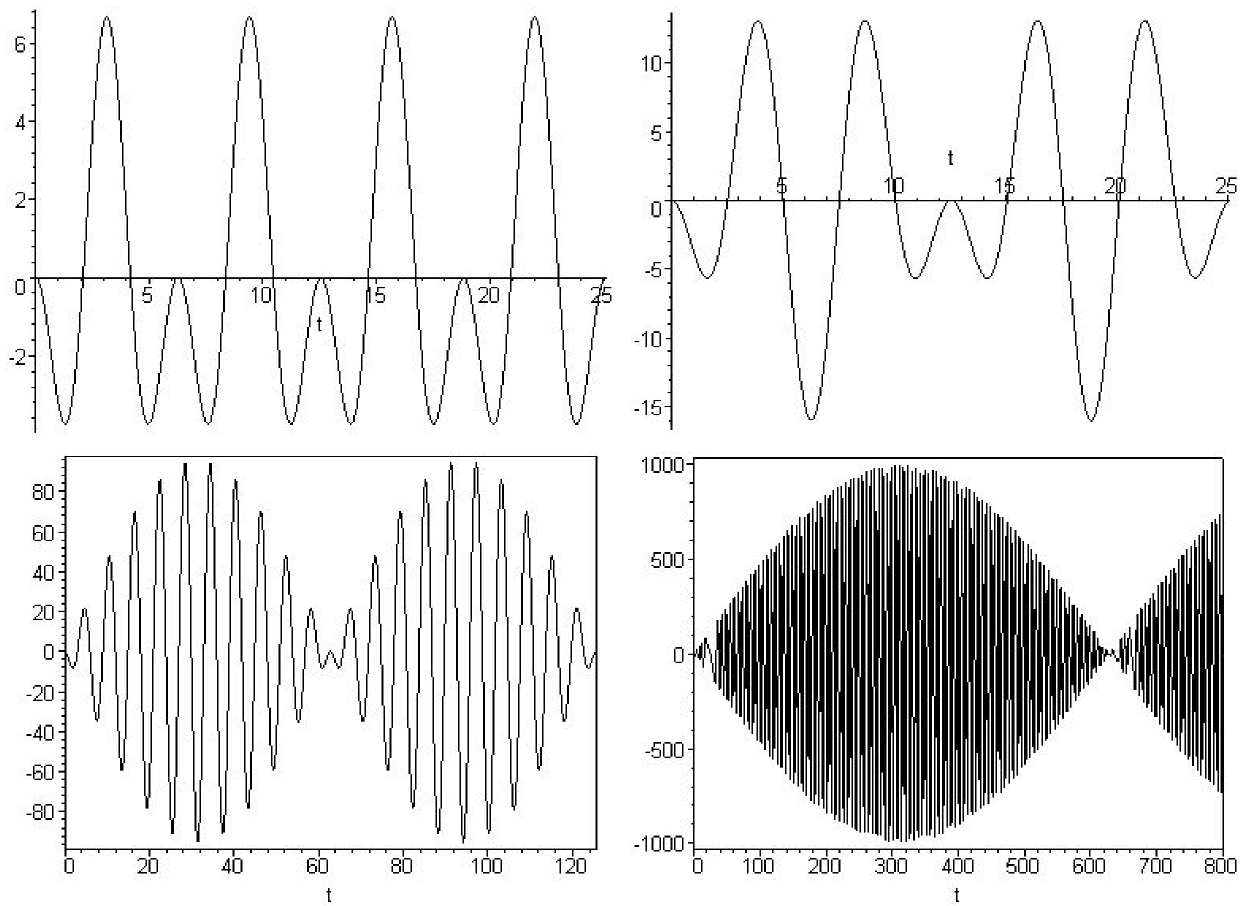

Figure 1: Figure showing the function $\frac{10}{1-w^2}\left(\cos(t)-\cos(wt)\right)$. The graphs show the result of taking $w = 2$, $w = 1.5$, $w = 1.1$ and $w = 1.01$. As $w \to 1$, we see that the amplitude and period of the beats are getting larger and larger and larger!

Homework
Replaces Section 3.9

1. Find the general solution of the given differential equation:

$$y'' + 3y' + 2y = \cos(t)$$

2. Pictured below are the graphs of several solutions to the differential equation:

$$y'' + by' + cy = \cos(\omega t)$$

Match the figure to the choice of parameters:

Choice	b	c	ω
(A)	5	3	1
(B)	1	3	1
(C)	5	1	3
(D)	1	1	3

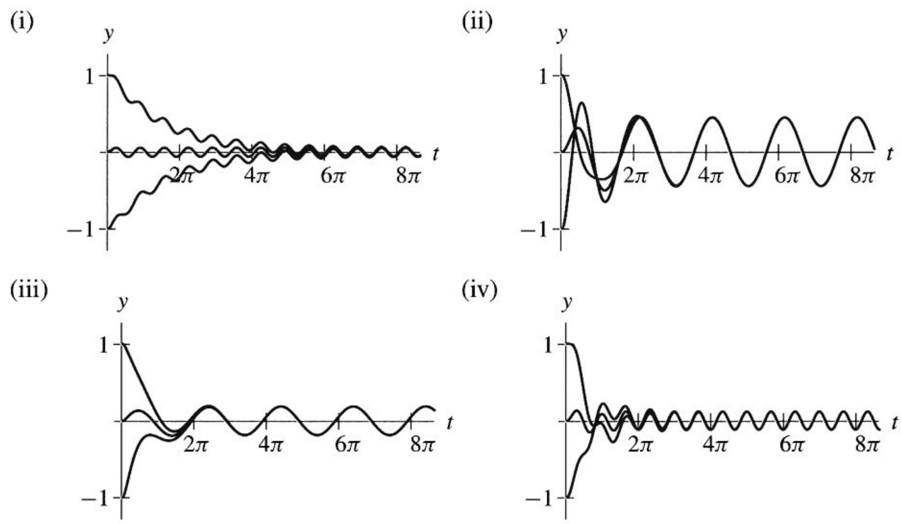

Figure 2: Figures for homework problem 2. Match each figure with the appropriate choice of constants.

3. Recall that

$$\text{Real}(e^{i\theta}) = \cos(\theta) \qquad \text{Imag}(e^{i\theta}) = \sin(\theta)$$

Show that, given the DE below we can use the ansatz $y_p = Ae^{3ti}$ (the real part),

$$y'' + 4y = 2\cos(3t)$$

and we will get the particular solution,

$$A = -\frac{2}{5} \qquad \Rightarrow \qquad y_p(t) = -\frac{2}{5}\cos(3t)$$

4. Fill in the question marks with the correct expression:

   Given the undamped second order differential equation, $y'' + \omega_0^2 y = A\cos(\omega t)$, we see "beating" if ?????????????. In particular, the longer period wave has a period that gets ?????????????? as $\omega \to \omega_0$, and its amplitude gets ?????????????

5. Find the solution to $y'' + 9y = 2\cos(3t)$, $y(0) = 0$, $y'(0) = 0$ by first solving the more general equation: $y'' + 9y = 2\cos(at)$, $y(0) = 0$, $y'(0) = 0$, then take the limit of your solution as $a \to 3$.

6. Suppose a unit mass is attached to a spring, with spring constant $k = 16$. Assuming that damping is negligible ($\gamma = 0$), suppose that we lightly tap the mass with a hammer (downward) every T seconds.

   Suppose the first tap is at $t = 0$, and before that time the mass is at rest[1]. Describe what you think will happen to the motion of the mass for the following choices of the tapping period T: (a) $T = \pi/2$ (b) $T = \pi/4$

   (HINT: Draw a picture of the homogeneous equation)

7. (Extra Practice) Can the following functions be linearly independent solutions to a second order linear homogeneous differential equation? Why or why not?

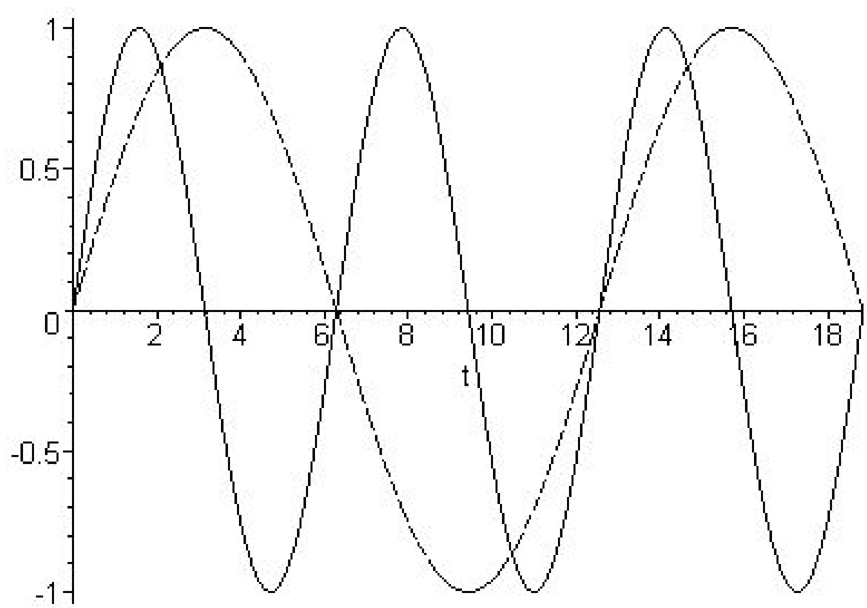

Figure 3: Can these functions be linearly independent solutions to a second order linear homogeneous differential equation?

[1]If you want to algebraically describe this, use initial conditions $u(0) = 0$ and $u'(0) = 1$. We will solve this problem completely after Spring Break.