# Sample Questions, Exam 1

Math 244

Remember, on the exam you may use a calculator, but NOT one that can perform symbolic manipulation (remembering derivative and integral formulas are a part of the course).

These questions are presented to give you an idea of the variety and style of question that will be on the exam. It is not meant to be exhaustive, so be sure that you understand the homework problems and quizzes.

## 1 Short Answer/True or False

1. True or False, and explain: If $y' = y + 2t$, then $0 = y + 2t$ is an equilibrium solution.
2. State the Existence and Uniqueness Theorem for linear first order initial value problems (IVPs).
3. State the general Existence and Uniqueness Theorem for first order initial value problems (IVPs).
4. Let $y' = f(y)$. It is possible to have two stable equilibrium with no other equilibrium between them.
5. What is a *linear* first order differential equation.
6. Let $y' = \sin(y)$. It is possible that the solution can oscillate (or be periodic).
7. What is an $n^{\text{th}}$ order differential equation?
8. What's the difference between:
   "The domain of $y(t)$" and "The time interval for which $y(t)$ is a solution to the DE"?
9. Let $\frac{dy}{dt} = 1 + y^2$. Then the solution will be valid for all $t$.
10. We said earlier that, under certain circumstances, solution curves to $y' = f(t, y)$ cannot cross (or be tangent) in the direction field. What are the conditions?
11. Problems 15-20 on page 8 (these are graphical questions- Be sure you look these over!).

## 2 Solve:

Give the general solution if there is no initial value. Before giving the solution, state what kind of differential equation it is (linear, separable, exact)- Multiple classes are possible, just give the one you will use.

1. $\dfrac{dy}{dx} = \dfrac{x^2 - 2y}{x}$
2. $(x + y)\,dx - (x - y)\,dy = 0$. Hint: Let $v = y/x$.
3. $\dfrac{dy}{dx} = \dfrac{2x + y}{3 + 3y^2 - x} \qquad y(0) = 0$.
4. $\dfrac{dy}{dx} = -\dfrac{2xy + y^2 + 1}{x^2 + 2xy}$
5. $\frac{dy}{dt} = 2\cos(3t) \qquad y(0) = 2$
6. $y' - \frac{1}{2}y = 0 \qquad y(0) = 200$. State the interval on which the solution is valid.
7. $y' - \frac{1}{2}y = e^{2t} \qquad y(0) = 1$
8. $y' = \frac{1}{2}y(3 - y)$
9. $\sin(2t)\;dt + \cos(3y)\;dy = 0$

10. $y' = xy^2$

11. $\dfrac{dy}{dt} = \mathrm{e}^{t+y}$

12. $\dfrac{dy}{dx} + y = \dfrac{1}{1+\mathrm{e}^x}$

13. $(t^2y + ty - y)\,dt + (t^2y - 2t^2)\,dy = 0$

14. $2xy^2 + 2y + (2x^2y + 2x)y' = 0$

15. $x^3\dfrac{dy}{dx} = 1 - 2x^2y.$

# 3 Misc.

1. Construct a linear first order differential equation whose general solution is given by:

$$y(t) = t - 3 + \frac{C}{t^2}$$

2. Construct a linear first order differential equation whose general solution is given by:

$$y(t) = 2\sin(3t) + C\mathrm{e}^{-2t}$$

3. Construct an autonomous differential equation that has stable equilibria at $y(t) = 1$ and $y(t) = 3$, and one unstable equilibrium at $y(t) = 2$. (Hint: Draw the phase plot first).

4. Construct an autonomous differential equation so that all solutions tend towards $y(t) = 2$ as $t \to \infty$.

5. Suppose we have a tank that contains M gallons of water, in which there is $Q_0$ pounds of salt. Liquid is pouring into the tank at a concentration of $r$ pounds per gallon, and at a rate of $\gamma$ gallons per minute. The well mixed solution leaves the tank at a rate of $\gamma$ gallons per minute.

   Write the initial value problem that describes the amount of salt in the tank at time $t$, and solve:

6. Referring to the previous problem, if let let the system run infinitely long, how much salt will be in the tank? Does it depend on $Q_0$? Does this make sense?

7. Modify problem 5 if: $M = 100$ gallons, $r = 2$ and the input rate is 2 gallons per minute, and the output rate is 3 gallons per minute. Solve the initial value problem, if $Q_0 = 50$.

8. Suppose an object with mass of 1 kg is dropped from some initial height. Given that the force due to gravity is 9.8 meters per second squared, and assuming a force due to air resistance of $\frac{1}{2}v$, find the initial value problem (and solve it) for the velocity at time $t$.

9. (Continuing with the last problem): At $t = 10$ minutes, the force due to air resistance suddenly changes to $10v$. Model the velocity for $t \geq 10$ (set up and solve the IVP):

10. (Continuing with the falling object): In a direction field, draw a sketch of the solution. HINT: These are autonomous differential equations, so you should draw the phase plot first!

11. Suppose $y' = ky(1-y)$, with $k > 0$. Sketch the phase diagram. Find and classify the equilibrium. Draw a sketch of $y$ on the direction field, paying particular attention to where $y$ is increasing/decreasing and concave up/down. Finally, get the analytic (general) solution.