Exercises: Nonlinear Analysis

1. For each system below, (i) Find all equilibrium solutions, (ii) Linearize the DE about each, and (iii) Use the Poincaré Diagram to analyze the stability. Finally, go online to the phase plane plotter and see if your analysis is correct.

   (a) $\begin{aligned} x' &= 1-y \\ y' &= x^2-y^2 \end{aligned}$ (b) $\begin{aligned} x' &= \cos(y) \\ y' &= \sin(x) \end{aligned}$ (c) $\begin{aligned} x' &= (2+x)(y-x) \\ y' &= (4-x)(y+x) \end{aligned}$

2. Consider the following three systems, and label them as A, B, and C:

   A. $\begin{aligned} x' &= x\left(\frac{3}{2}-x-\frac{1}{2}y\right) \\ y' &= y\left(2-y-\frac{3}{4}x\right) \end{aligned}$ B. $\begin{aligned} x' &= x\left(1-\frac{1}{2}x-\frac{1}{2}y\right) \\ y' &= y\left(-\frac{1}{4}+\frac{1}{2}x\right) \end{aligned}$ C. $\begin{aligned} x' &= x(1-x-y) \\ y' &= y\left(\frac{3}{2}-y-x\right) \end{aligned}$

   (a) Which systems correspond to a system of competing species? Which correspond to a predator-prey system? (Be sure to explain your reasoning).

   (b) In each of the competing species models, determine whether or not there will be peaceful coexistence or extinction. Hint: You only need to determine one equilibrium solution for each.

   (c) In the predator-prey model, do a complete analysis: Find all equilibria, linearize the system, then use the Poincaré Diagram to classify the equilibrium. Give a conclusion about the corresponding ecosystem- Will there be problems?

3. Sometimes nonlinear differential equations can be solved by means of methods from Chapter 2, where we convert the system using $dx/dt, dy/dt$ to dy/dx (we've already done a few of these). Here are some more (one linear one just for fun), as a start to your review for the final! Remember, each DE may be classified in multiple ways, so if you get stuck using one technique, you might try another.

   (a) $\begin{aligned} x' &= 2y-2 \\ y' &= -(2x+3) \end{aligned}$ (c) $\begin{aligned} x' &= x-y \\ y' &= y-4x \end{aligned}$

   (b) $\begin{aligned} x' &= y(1-x^3) \\ y' &= x^2 \end{aligned}$ (d) $\begin{aligned} x' &= \mathrm{e}^x\cos(y)+2\cos(x) \\ y' &= 2y\sin(x)-\mathrm{e}^x\sin(y) \end{aligned}$

4. (Optional) Here is an interesting system of equations:

   $$\begin{aligned} x' &= x+y-x(x^2+y^2) \\ y' &= -x+y-y(x^2+y^2) \end{aligned}$$

   Go online to the phase plane plotting website and put these equations in. Draw several solutions to the differential equation, and you should see something very familiar- Something you've probably been looking at all semester long!