Exercise Set 3 (HW for 7.6, 7.8)

In this homework set, we will practice finding eigenvalues and eigenvectors when the eigenvalues are either complex or the matrix is defective.

1. Given a 2×2 defective matrix A with double eigenvalue λ, eigenvector $\mathbf{v}$ and generalized eigenvector $\mathbf{w}$, show that the function:

$$\mathrm{e}^{\lambda t}(t\mathbf{v} + \mathbf{w})$$

   solves the differential equation $\mathbf{x}' = A\mathbf{x}$.

2. Exercises 1, 3, pg. 409 (Section 7.6, solve with complex evals/evecs)

3. Exercises 1, 3, 7, pg. 429 (Section 7.8, solve with degenerate matrix)

4. Exercises 13, 15, pg 410 (Section 7.6, try to analyze with parameter- We'll do these more in depth later as well).

5. Given the eigenvalues and eigenvectors for some matrix A, write the general solution to $\mathbf{x}' = A\mathbf{x}$. Furthermore, classify the origin as a sink, source, spiral sink, spiral source, saddle, or none of the above.

   (a) $\lambda = -1 + 2i \qquad \mathbf{v} = \begin{bmatrix} 1-i \\ 2 \end{bmatrix}$

   (b) $\lambda = -2, 3 \qquad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (c) $\lambda = -2, -2 \qquad \mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

   (d) $\lambda = 2, -3 \qquad \mathbf{v}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

   (e) $\lambda = 1 + 3i \qquad \mathbf{v} = \begin{bmatrix} 1 \\ 1-i \end{bmatrix}$

   (f) $\lambda = 2i \qquad \mathbf{v} = \begin{bmatrix} 1+i \\ 1 \end{bmatrix}$

6. Use the Poincaré Diagram on page 497, where $p = \mathrm{Tr}(A)$ and $q = \det(A)$ to determine the stability of the origin for $\mathbf{x}' = A\mathbf{x}$, if A is given below:

   (a) $\begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} -\frac{1}{2} & 1 \\ -1 & -\frac{1}{2} \end{bmatrix}$

   (c) $\begin{bmatrix} -1 & -1 \\ 0 & -\frac{1}{4} \end{bmatrix}$

   (d) $\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$