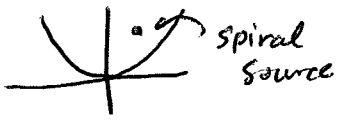
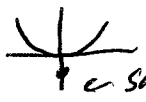
HW Solns, Linear Systems

1. (a) $\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$ $\quad Tr(A) = 2$, $\det(A) = 5$, $\Delta = 4 - 20 = -16$



Soln: $c_1 e^t \begin{bmatrix} \cos(2t) \\ \cos(2t) + \sin(2t) \end{bmatrix} + c_2 e^t \begin{bmatrix} \sin(2t) \\ \sin(2t) - \cos(2t) \end{bmatrix}$

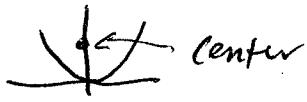
(b) $\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$ $\quad Tr(A) = 0$, $\det(A) = -1$



(don't need Δ)

Soln: $c_1 e^{-t} \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ $\quad Tr(A) = 0$, $\det(A) = 4$, $\Delta = -16$

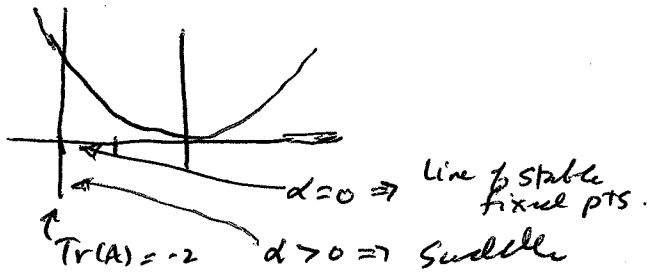


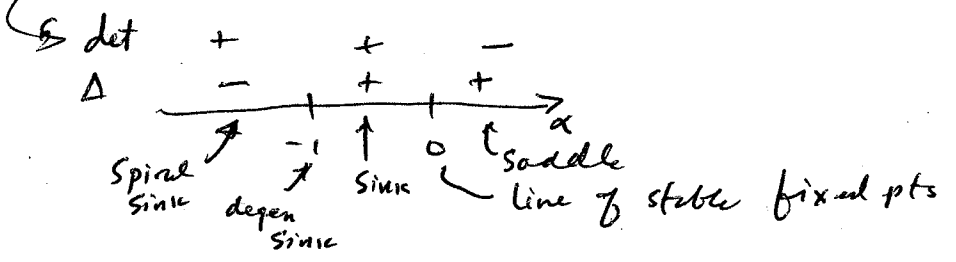
Soln: $c_1 \begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} -\cos(2t) \\ \sin(2t) \end{bmatrix}$

(d) [Also done in class] $\begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix}$ $\quad Tr(A) = 0$, $\det(A) = 0$, $\Delta = 0$ "Uniform motion"

$$c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \left[t \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1/4 \end{bmatrix} \right]$$

2 (a) $\begin{bmatrix} 0 & \alpha \\ 1 & -2 \end{bmatrix}$ $\quad Tr(A) = -2$, $\det(A) = -\alpha$, $\Delta = 4 + 4\alpha$

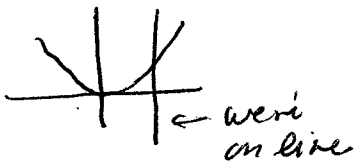


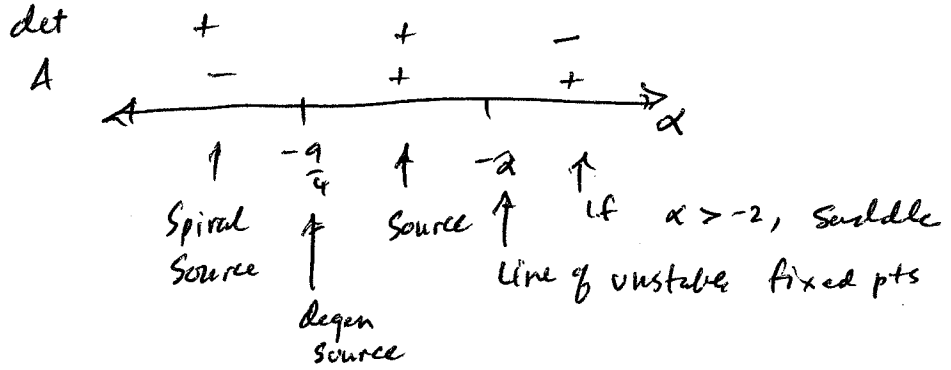


2(b) $\begin{bmatrix} 2 & \alpha \\ 1 & -1 \end{bmatrix}$ $\quad Tr(A) = 1$

$\det(A) = -(2+\alpha)$

$\Delta = 1 + 4(2+\alpha) = 9 + 4\alpha$





2(c): Quiz

3(a) Using the first eqn, $x(1-y) = 0$ or $x = 0$ or $y = 1$

Now, if $x = 0$:

Then $y = 0$ in the 2nd eqn.

If $y = 1$

Then $1 + 2x = 0$ or $x = -1/2$

$\Rightarrow (0,0)$ & $(-1/2, 1)$

(b) From 1st eqn, $y = 0$ or $y = 2 - x$

If $y = 0$, then 2nd eqn: $x = 0$

If $y = 2 - x$, 2nd eqn: $-x - (2-x) - 2x(2-x) = 0$

or $x = 1 \pm \sqrt{2}$

$\Rightarrow (0,0), (1+\sqrt{2}, 1-\sqrt{2}), (1-\sqrt{2}, 1+\sqrt{2})$

(c) $x = \pm 1/\sqrt{3}$, $y = -1/2$.