Extra Practice: Section 2.5

1. Given the differential equation, identify if each is *linear* (L), *separable* (S), *autonomous* (A), *Bernoulli* (B), and/or *homogeneous* (H). Recall that any given DE may have multiple labels.

   (a) $\dfrac{dy}{dx} = \dfrac{x^3 - 2y}{x}$

   (b) $\dfrac{dy}{dx} = \dfrac{x + y}{x - y}$

   (c) $(e^x + 1)\dfrac{dy}{dx} = y - ye^x$

   (d) $\dfrac{dy}{dt} = \cos(y)$

   (e) $\dfrac{dy}{dt} = \cos(t)$

   (f) $t^2 y' + 2ty - y^3 = 0$

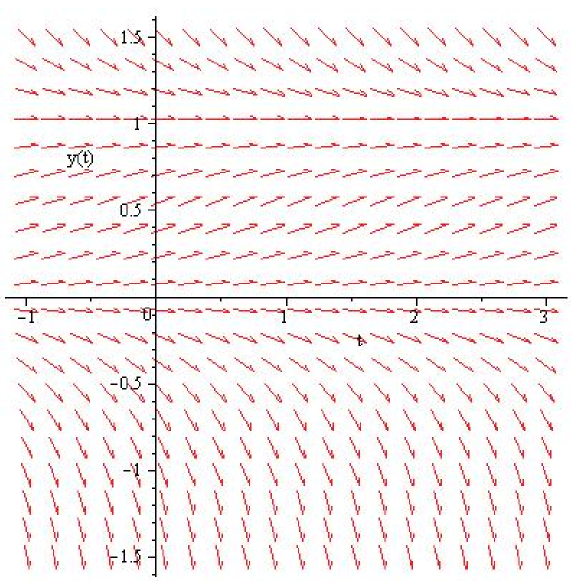
2. Suppose we are given the differential equation:

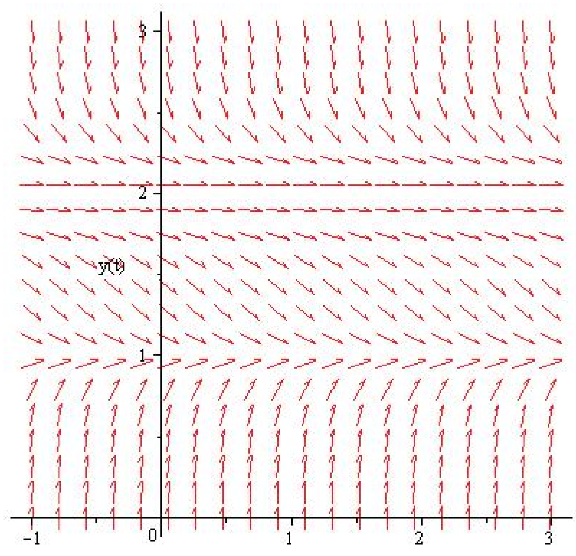
$$y' = \sin(y)$$

   (a) True or False: The solution may be periodic.

   (b) What happens to the solution corresponding to $y(0) = 1$? How about $y(0) = 100$? (HINT: Do not solve!)

3. Below are two direction fields (in the (t, y) plane). Find an autonomous differential equation, $y' = F(y)$, that is consistent with each one. Proceed by first sketching a consistent function for each direction field in the (y, y') plane.





4. Let $y' = y(y-1)$.

   (a) Give the general solution.

   (b) Plot the appropriate function in the (y, y') plane, and classify the equilibria as to stability.

   (c) Without going to the solution $y(t)$, find intervals on which $y(t)$ will be concave up and concave down.

5. For each fraction, write down what the partial fraction expansion would be (but do not solve for the constants!):

   (a) $\dfrac{x^2 - 3x + 1}{x(x-1)(x-2)}$

   (b) $\dfrac{3x - 1}{x^2(x-1)}$

   (c) $\dfrac{3x - 1}{(x+1)(x^2 + 2x + 3)}$

   (d) $\dfrac{3x - 1}{(x^2+1)(x^2+4)^2}$