3.1

#1 Ansatz: $y = e^{rt} \Rightarrow r^2 + 2r - 3 = 0$.

$$\Rightarrow (r-1)(r-2) = 0$$
$$r = 1, 2$$
$$\Rightarrow y = c_1 e^t + c_2 e^{2t}$$

#3: $6r^2 - r - 1 = 0 \Rightarrow (3r+1)(2r-1) = 0$

$$\Rightarrow r = -1/3, \ 1/2$$
$$y = c_1 e^{-t/3} + c_2 e^{t/2}$$

#10 Solve the IVP: $y'' + 4y' + 3y = 0, \ y(0) = 2, \ y'(0) = -1$

(a) Char eqn: $r^2 + 4r + 3 = 0 \Rightarrow (r+3)(r+1) = 0$

so $y = c_1 e^{-3t} + c_2 e^{-t}$ (and $y' = -3c_1 e^{-3t} - c_2 e^{-t}$)

solving for c_1 & c_2:

($y(0) = 2$) $\quad 2 = c_1 + c_2$

$-1 = -3c_1 - c_2 \Rightarrow$

$$c_1 = \frac{\begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -3 & -1 \end{vmatrix}} \qquad c_2 = \frac{\begin{vmatrix} 1 & 2 \\ -3 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -3 & -1 \end{vmatrix}}$$

$$c_1 = \frac{-2+1}{-1+3} = -1/2 \qquad c_2 = \frac{-1+6}{-1+3} = 5/2$$

$$\Rightarrow y = -\frac{1}{2} e^{-3t} + \frac{5}{2} e^{-t}.$$

(same technique for 15, 16) $\Big\}$ Because both exponentials have neg constants, we see that $y \to 0$ as $t \to \infty$ (for the sketch)

3.1, cont.

#17 Work backwords from the characteristic eqn.

That is, $ay'' + by' + cy = 0 \longleftrightarrow ar^2 + br + c = 0$

So, if $r_1 = 2$ and $r_2 = -3$, then the char eqn is

$$(r-2)(r+3) = 0 \longleftrightarrow r^2 + r - 6 = 0$$

$$y'' + y' - 6y = 0$$

18 (Same idea as 17)

20. First, $2r^2 + 3r + 1 = 0 \Rightarrow r = 1/2, 1$

So $y = c_1 e^{t/2} + c_2 e^t$ (and $y' = \frac{c_1}{2} e^{t/2} + c_2 e^t$).

Solving the IVP, we get:

$$y = 3e^{t/2} - e^t$$

To find the max, set $y' = 0$:

$$y' = \frac{3}{2} e^{t/2} - e^t = 0 \Rightarrow t = \ln(9/4)$$

Using this, we find $y(\ln(9/4)) = \frac{9}{4}$.

To find where $y=0$,

$$3e^{t/2} - e^t = 0 \Rightarrow t = \ln$$

#21 See 22

#22 First, $4r^2 - 1 = 0 \Rightarrow r = \pm 1/2$

so $y = c_1 e^{t/2} + c_2 e^{-t/2}$ with $y(0) = 2$ and $y'(0) = \beta$,

then
$$\begin{aligned} c_1 + c_2 &= 2 \\ \tfrac{1}{2}c_1 - \tfrac{1}{2}c_2 &= \beta \end{aligned} \Rightarrow \begin{aligned} c_1 + c_2 &= 2 \\ c_1 - c_2 &= 2\beta \end{aligned}$$

so
$$c_1 = \frac{\begin{vmatrix} 2 & 1 \\ 2\beta & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} \qquad c_2 = \frac{\begin{vmatrix} 1 & 2 \\ 1 & 2\beta \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}}$$

$$c_1 = 1 + \beta \qquad c_2 = 1 - \beta$$

For $y \to 0$, the constant c_1 must be zero,

so that means $\beta = -1$,

#23, The roots to char eqn are $r = \alpha, \alpha - 1$.

- For all solns $\to 0$, both of these should be negative, or $\alpha < 0$.
- ~~Otherwise~~, If both are positive, all soln's $\nearrow \infty$, so $\alpha > 1$.

#24 (Same idea as 23)