Last time:

- Vocab: ODE, PDE, IVP
- Skills: Be able to verify that $\phi(t)$ is a solution to a DE.
- Given the general solution to $y' = ay + b$ (it is $y = P\mathrm{e}^{at} - \frac{b}{a}$)
- Three models: Free fall, Mice/Owls, Newton's Law of Cooling.

Today: Finish up visualizations in Chapter 1, look at an algorithm in 2.1.
First, some vocab from the reading:

- Order of a DE

Today: Finish up visualizations in Chapter 1, look at an algorithm in 2.1. First, some vocab from the reading:

- Order of a DE (order of highest derivative)

Today: Finish up visualizations in Chapter 1, look at an algorithm in 2.1.
First, some vocab from the reading:

- Order of a DE (order of highest derivative)
  Example: Order of $y' + y^3 = t^2 + 4t + 5$ is:

Today: Finish up visualizations in Chapter 1, look at an algorithm in 2.1.
First, some vocab from the reading:

- Order of a DE (order of highest derivative)
  Example: Order of $y' + y^3 = t^2 + 4t + 5$ is: 1

Today: Finish up visualizations in Chapter 1, look at an algorithm in 2.1.
First, some vocab from the reading:

- Order of a DE (order of highest derivative)
  Example: Order of $y' + y^3 = t^2 + 4t + 5$ is: 1
- Linear DE

Today: Finish up visualizations in Chapter 1, look at an algorithm in 2.1. First, some vocab from the reading:

- Order of a DE (order of highest derivative)
  Example: Order of $y' + y^3 = t^2 + 4t + 5$ is: 1
- Linear DE (linear in y, y', y'', etc)

Today: Finish up visualizations in Chapter 1, look at an algorithm in 2.1.
First, some vocab from the reading:

- Order of a DE (order of highest derivative)
  Example: Order of $y' + y^3 = t^2 + 4t + 5$ is: 1
- Linear DE (linear in y, y', y", etc)
  Example: $y' + y^3 = t^2 + 4t$ is

Today: Finish up visualizations in Chapter 1, look at an algorithm in 2.1.
First, some vocab from the reading:

- Order of a DE (order of highest derivative)
  Example: Order of $y' + y^3 = t^2 + 4t + 5$ is: 1
- Linear DE (linear in y, y', y", etc)
  Example: $y' + y^3 = t^2 + 4t$ is nonlinear (y^3)
  Example: $y'' + 3y' + 5y = 4t^2$ is

Today: Finish up visualizations in Chapter 1, look at an algorithm in 2.1.
First, some vocab from the reading:

- Order of a DE (order of highest derivative)
  Example: Order of $y' + y^3 = t^2 + 4t + 5$ is: 1
- Linear DE (linear in y, y', y", etc)
  Example: $y' + y^3 = t^2 + 4t$ is nonlinear (y^3)
  Example: $y'' + 3y' + 5y = 4t^2$ is linear (in y, y', etc).
  (More on this later)

Questions that we try to answer for DEs:

- *Existence*: Does every ODE $y' = f(t, y)$ have a solution $y = \phi(t)$?

Questions that we try to answer for DEs:

- *Existence*: Does every ODE $y' = f(t, y)$ have a solution $y = \phi(t)$? (No).
- A second question is one of *uniqueness*: If the ODE has a solution, does it have more than one?

Questions that we try to answer for DEs:

- *Existence*: Does every ODE $y' = f(t, y)$ have a solution $y = \phi(t)$? (No).
- A second question is one of *uniqueness*: If the ODE has a solution, does it have more than one?
- Third is a practical question: If the ODE has a solution, can we compute it?

Questions that we try to answer for DEs:

- *Existence*: Does every ODE $y' = f(t, y)$ have a solution $y = \phi(t)$? (No).
- A second question is one of *uniqueness*: If the ODE has a solution, does it have more than one?
- Third is a practical question: If the ODE has a solution, can we compute it?

TODAY: Visualizing solutions, solving a linear equation.

Visualizing solutions to DE

Visualizing solutions to DE

$$y' = ay + b \qquad y(t) = P_0 e^{at} - \frac{b}{a}$$

Cases:

- If $P_0 = 0$, then $y(t)$ is constant $(y = -b/a)$.
  Definition: An **equilibrium solution** is a constant solution $y = k$ so that $y' = 0$.
- Otherwise:
  - If $a > 0$, then the solutions will all "blow up" $(|y(t)| \to \infty)$ except one solution.
  - If $a < 0$, then all solutions tend toward equilibrium.

Visualizing Solutions

A differential equation is like a "road map":

$$y' = f(t, y)$$

That is, at each point (t, y), we can compute the slope of the line tangent to the solution curve $y(t)$.

Visualizing Solutions

A differential equation is like a “road map”:

$$y' = f(t, y)$$

That is, at each point (t, y), we can compute the slope of the line tangent to the solution curve $y(t)$.
If the function y is well behaved, the tangent line should be a good approximation to y.

Visualizing Solutions

A differential equation is like a "road map":

$$y' = f(t, y)$$

That is, at each point (t, y), we can compute the slope of the line tangent to the solution curve $y(t)$.
If the function y is well behaved, the tangent line should be a good approximation to y.

Definition: A direction field is a plot in the (t, y) plane that give the local tangent lines to the solution to a first order ODE.

Example: $y' = t - y^2$

Visualizing Solutions

A differential equation is like a “road map”:

$$y' = f(t, y)$$

That is, at each point (t, y), we can compute the slope of the line tangent to the solution curve $y(t)$.
If the function y is well behaved, the tangent line should be a good approximation to y.

Definition: A direction field is a plot in the (t, y) plane that give the local tangent lines to the solution to a first order ODE.

Example: $y' = t - y^2$
In drawing a picture, we might consider curves of constant slope. For example, with zero slope:

$$0 = t - y^2 \quad \Rightarrow \quad y^2 = t$$

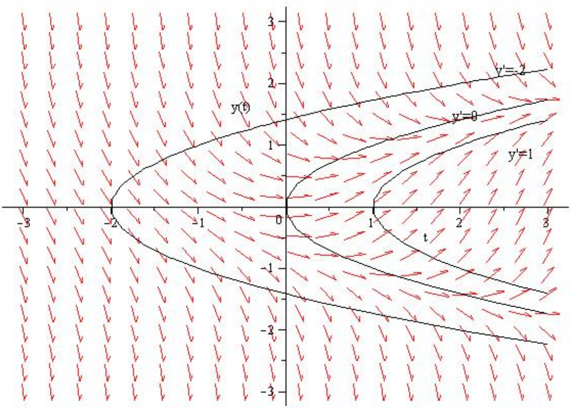
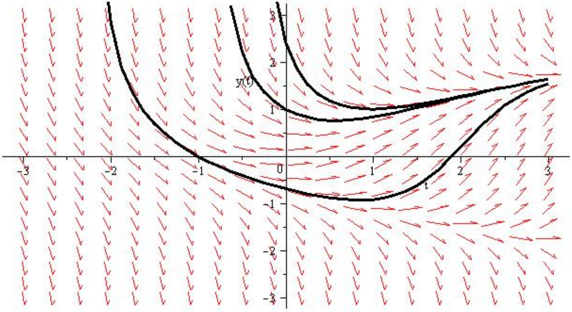
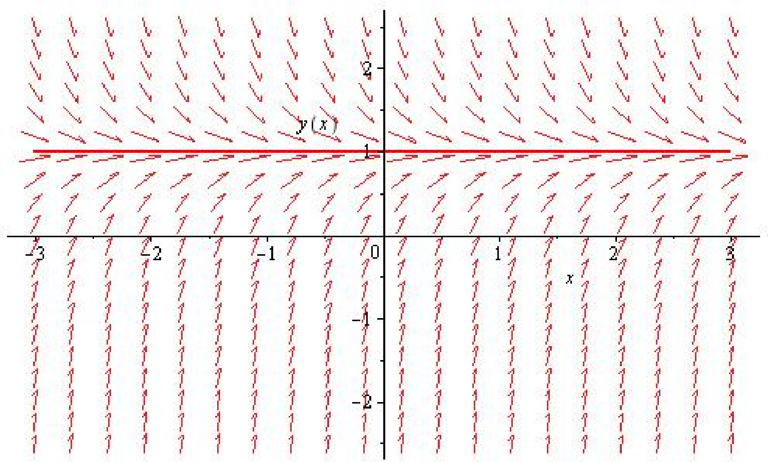


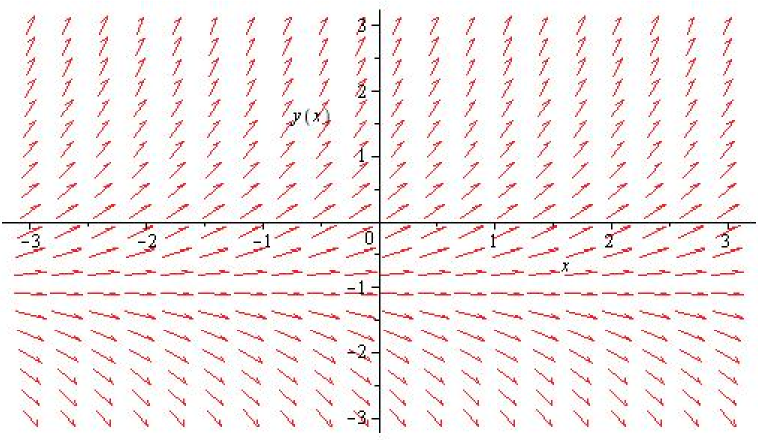
Figure: Direction Field with Isoclines: $y' = -2, y' = 0, y' = 1$


y(t)
t
-3
-2
-1
0
1
2
3

Give an ODE of the form $y' = ay + b$ whose direction field looks like:

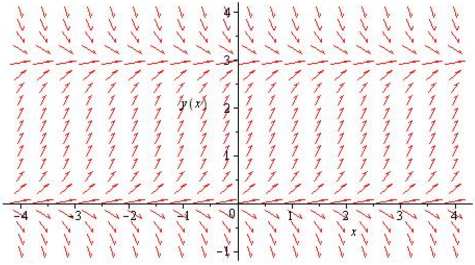


Same question as before:



Choose a DE

1. $y' = 3 - y$
2. $y' = y(y + 3)$
3. $y' = y(3 - y)$
4. $y' = 2y - 1$



Homework Hint: #22, Section 1.1

$$V = \frac{4}{3}\pi r^3 \qquad A = 4\pi r^2$$

so if $V' = kA$, give V' in terms of V only.

Homework Hint: #14, Section 1.3

Differentiate the following with respect to t:

$$f(t) \int_0^t G(s)\, ds$$

SOLUTION:

$$f'(t) \int_0^t G(s)\, ds + f(t)G(t)$$

Section 2.1: Linear DEs

Definition: Linear first order is

$$y' + a(x)y = f(x) \qquad\qquad y' + a(t)y = f(t)$$

Solution is by first computing an "integrating factor".
We'll do this on the board.