Solutions to Selected Problems, 3.7 (Model of Mass-Spring System)

NOTE about units: On quizzes/exams, we will always use the standard units of meters, kilograms and seconds, or feet, pounds and seconds. The textbook likes to mix them up somewhat.

5. A mass weighing 2 lb stretches a spring 6 inches.

   Remark: This information is here so that we can get the spring constant. Change the 6 inches to 1/2 foot:

$$mg - kL = 0 \quad \Rightarrow 2 - \frac{k}{2} = 0 \quad \Rightarrow \quad k = 4$$

   From this, we can also get the mass using $g = 32$ ft/sec^2 (the constant would be given to you):

$$mg = 2 \quad \Rightarrow \quad m = \frac{2}{g} = \frac{2}{32} = \frac{1}{16}$$

   Continuing with the problem, we only need to determine γ- Since there is no damping, $\gamma = 0$, and

$$\frac{1}{16}u'' + 0u' + 4u = 0 \quad \Rightarrow \quad u'' + 64u = 0$$

   If the mass is pulled down 3 inches and released, the initial conditions are $u(0) = \frac{1}{4}$ and $u'(0) = 0$. Solving the IVP, we get

$$u(t) = \frac{1}{4}\cos(8t)$$

   so the amplitude is 1/4 and the period is $2\pi/8$ or $\pi/4$. You don't need to plot it for now.

7. Very similar to 5.- For the exam, I will not ask you to determine the frequency, period amplitude, phase of the motion (but be able to solve the IVP).

9. This one is tricky in terms of the units (See the note at the top), but if we continue, we will write cm rather than meters, and 9.8 becomes 980. The spring constant:

$$k = \frac{20 \cdot 980}{5} = 3920 \text{ dyne/cm}$$

   And the IVP:

$$20u'' + 400u' + 3920u = 0 \qquad u(0) = 2, u'(0) = 0$$

   (where u is measured in cm and time is in seconds).

11. (Watch the units!) Building the model, the spring constant is

$$k = \frac{3}{0.1} = 30 \text{ N/m}$$

and the damping coefficient:

$$\gamma u' = F_d \quad \Leftrightarrow \gamma(5) = 3 \quad \Rightarrow \gamma = \frac{3}{5}$$

so that

$$2u'' + \frac{3}{5}u' + 30u = 0$$

(The numbers are a little messy in the solution).

24. $\frac{3}{2}u'' + ky = 0$ with $u(0) = 2$ and $u'(0) = v$.

SOLUTION: For the standard model with no damping, we know that the circular period of the homogeneous part of the solution is (also see pg. 195):

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{2k}{3}}$$

In this case, if the (regular) period is π, then we substitute and solve:

$$\pi = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{2k/3}} \qquad \Rightarrow \qquad k = 6$$

Solving the DE in terms of v, we get:

$$u(t) = 2\cos(2t) + \frac{v}{2}\sin(2t)$$

so the amplitude of the solution is

$$A = \sqrt{2^2 + \left(\frac{v}{2}\right)^2} = \frac{\sqrt{16 - v^2}}{2}$$

Setting this equal to 3 gives us the solution for v:

$$v = \pm 2\sqrt{5}$$