Selected Solutions to 3.8

In Exercises 1-3, use the formulas given in the text to re-write the function as a product. You do not need to memorize them. For example, the solution to 3 is given below:

3. $\cos(\pi t) + \cos(2\pi t)$

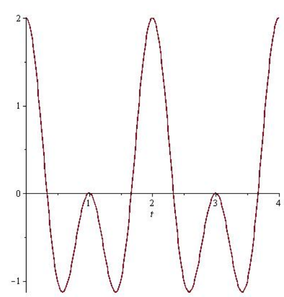
   SOLUTION: The formula is

$$\cos(A) + \cos(B) = 2\cos\left(\frac{B-A}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

   Therefore,

$$2\cos\left(\frac{\pi}{2}t\right)\cos\left(\frac{3\pi}{2}t\right)$$

   Side Remark: One cosine is "slow" and the other is "fast", which one can see in the figure:



6. From what is given, $m = 5$. To find the spring constant, express the length in meters (g is in meters): $L = 0.1$ meters. Now, set $mg - kL = 0$ (and $g = 9.8$):

$$5 \cdot 9.8 - k(0.1) = 0 \qquad \Rightarrow \qquad k = (10)(5)(9.8) = 98 \cdot 5 = 490$$

   If the force is 2 when speed is 0.04 meters per second, then $\gamma v' = 2$, or $\gamma \cdot 0.04 = 2$, or

$$\gamma = 50$$

   Therefore,

$$5u'' + 50u' + 490u = 10\sin\left(\frac{t}{2}\right), \quad u(0) = 0, u'(0) = 0.03$$

11. For these units to be consistent, let's stick with feet, lbs and seconds.

   We're given that a mass weighing 8 lbs stretches a spring 6 inches (1/2 feet). We can get the spring constant from this (recall that $mg = 8$):

$$mg - kL = 0 \qquad \Rightarrow \qquad 8 - \frac{k}{2} = 0 \qquad \Rightarrow \qquad k = 16$$

And the value of m (I would tell you that $g = 32$):

$$m \cdot 32 = 8 \quad \Rightarrow \quad m = \frac{8}{32} = \frac{1}{4}$$

We're told that the damping constant is $\gamma = 0.25$. Now we have our differential equation:

$$\frac{1}{4}u'' + \frac{1}{4}u' + 16u = 4\cos(2t) \qquad \text{or} \qquad u'' + u' + 64u = 16\cos(2t)$$

The steady state solution is actually the particular solution (I won't use the term steady state on the exam, but you should be able to find the particular solution): Using the Method of Undetermined Coefficients,

$$u_p(t) = A\cos(2t) + B\sin(2t)$$

Put this into the DE and solve for A, B:

$$\begin{array}{rlll} 64u_p = & 64A\cos(2t) & +64B\sin(2t) \\ +u_p' = & -2B\cos(2t) & +2A\sin(2t) \\ +u_p'' = & -4A\cos(2t) & -4B\sin(2t) \\ \hline 16\cos(2t) = & (60A - 2B)\cos(2t) & +(60B + 2A)\sin(2t) \end{array} \quad \Rightarrow \quad \begin{array}{rl} 30A - B & = 8 \\ -A + 30B & = 0 \end{array}$$

Where I simplified a bit by dividing both equations by 2.

Therefore, by Cramer's Rule, first we compute the determinant of the coefficient matrix: $30^2 + 1 = 901$.

$$A = \frac{\begin{vmatrix} 8 & -1 \\ 0 & 30 \end{vmatrix}}{901} = \frac{240}{901} \qquad B = \frac{\begin{vmatrix} 30 & 8 \\ -1 & 0 \end{vmatrix}}{901} = \frac{8}{901}$$

Therefore,

$$u_p(t) = \frac{240}{901}\cos(2t) + \frac{8}{901}\sin(2t)$$

Side Remark: Sorry about the messy fractions!

NOTE: We won't do part (b) of the question, since we didn't quite get there.

13. We didn't discuss damped forced equations in class, so this won't be on the exam.