Math 244
Homework To Replace 7.2

1. Exercise 22 (Section 7.1, p 363)

2. Give the solution to each system. If it has an infinite number of solutions, give your answer in vector form:

$$\begin{aligned} 3x+2y &= 1 \\ 2x-y &= 3 \end{aligned} \qquad \begin{aligned} 3x+2y &= 1 \\ 6x+4y &= 3 \end{aligned} \qquad \begin{aligned} 3x+2y &= 1 \\ 6x+4y &= 2 \end{aligned}$$

3. Write each of the previous systems in matrix-vector form. Verify that the determinant of the first matrix is not zero, but is zero for the second and third.

4. Write each system of differential equations in matrix-vector form or write the system from the matrix-vector form:

$$\begin{aligned} x_1' &= 3x_1 - x_2 \\ x_2' &= 9x_1 - 3x_2 \end{aligned} \qquad \mathbf{x}' = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} \mathbf{x}$$

5. Find the equilibrium solutions to the previous autonomous linear differential equations.

   For the following problems, define

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

6. Similar to Exercise 1 (Section 7.2). Given the matrices A and B above:

7. Compute $A - 4B$
8. Compute $(B - 4I)\mathbf{b}$
9. Compute $\det(B - 4I)$
10. Are AB and BA the same?
11. Compute $A^{-1}\mathbf{b}$
12. Verify: $A\mathbf{b} - 3\mathbf{b} = (A - 3I)\mathbf{b}$
13. Compute $B^T B$, $\mathrm{Tr}(A)$, and $\mathrm{Tr}(B)$
14. Is $A^T = A$? Is $B^T = B$?

15. Solve the system of complex equations. Before we start, check that we get the bottom equation if we divide the top equation by $(1+i)$.

$$\begin{aligned} -(1+i)v_1 \quad &+2v_2 &= 0 \\ -v_1 \quad &+(1-i)v_2 &= 0 \end{aligned}$$

16. Solve the system of complex equations. Before we start, check that we get the bottom equation if we divide the top equation by $(2-i)$.

$$\begin{aligned} (2-i)v_1 \quad &-5v_2 &= 0 \\ v_1 \quad &-(2+i)v_2 &= 0 \end{aligned}$$

17. Exercises 22, 23 (Section 7.2, p. 373)

    (a) If $\mathbf{x}' = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} \mathbf{x}$, verify that a solution is $\mathbf{x}(t) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} e^{2t}$

    (b) If $\mathbf{x}' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{t}$, verify that a solution is $\mathbf{x}(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{t} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} te^{2t}$