Exercise Set 4 (HW for 7.6, 7.8)

1. Solve $\mathbf{x}' = A\mathbf{x}$, where A is given below:

   (a) $A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$

   (d) $A = \begin{bmatrix} -3/2 & 1 \\ -1/4 & -1/2 \end{bmatrix}$

   (e) $A = \begin{bmatrix} 1 & -1 \\ 5 & -3 \end{bmatrix}$

2. For 1(a) and 1(b), convert to a second order differential equation and solve by methods from Chapter 3.

3. Given the eigenvalues and eigenvectors for some matrix A, write the general solution to $\mathbf{x}' = A\mathbf{x}$. Furthermore, classify the origin as a sink, source, spiral sink, spiral source, saddle, or none of the above.

   (a) $\lambda = -1 + 2i \qquad \mathbf{v} = \begin{bmatrix} 1-i \\ 2 \end{bmatrix}$

   (b) $\lambda = -2, 3 \qquad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (c) $\lambda = -2, -2 \qquad \mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

   (d) $\lambda = 2, -3 \qquad \mathbf{v}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

   (e) $\lambda = 1 + 3i \qquad \mathbf{v} = \begin{bmatrix} 1 \\ 1-i \end{bmatrix}$

   (f) $\lambda = 2i \qquad \mathbf{v} = \begin{bmatrix} 1+i \\ 1 \end{bmatrix}$

4. (Also see 13-20, p. 410) For each system $\mathbf{x}' = A\mathbf{x}$, the matrix A will depend upon the parameter α: (i) Determine the eigenvalues in terms of α, (ii) Find the critical values of α where the behavior of the solution to the system changes significantly. We'll go through one or two in class.

   (a) $\begin{bmatrix} \alpha & 1 \\ -1 & \alpha \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & -5 \\ 1 & \alpha \end{bmatrix}$

   (c) $\begin{bmatrix} 2 & -5 \\ \alpha & -2 \end{bmatrix}$

   (d) $\begin{bmatrix} 3 & \alpha \\ -6 & -4 \end{bmatrix}$