Exercise Set 5 (Finishing Chapter 7)

1. Solve $\mathbf{x}' = A\mathbf{x}$, where A is given below. Also, classify the origin using the Poincaré diagram.

   (a) $A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$

2. Solve the following second order IVP three ways: (i) Using methods of Chapter 3, (ii) Using the Laplace transform, and (iii) by converting it into a system of first order.

$$y'' - 2y' - 3y = 0, \quad y(0) = 1, y'(0) = 1$$

3. For the following *nonlinear* systems, find the equilibrium solutions, then find the general solution by looking at dy/dx:

$$\begin{aligned} x' &= x - xy \\ y' &= y + 2xy \end{aligned}$$

4. For each system $\mathbf{x}' = A\mathbf{x}$, the matrix A will depend upon the parameter α: (i) Determine the eigenvalues in terms of α, (ii) Find the critical values of α where the behavior of the solution to the system changes significantly. We'll go through one or two in class.

   (a) $\begin{bmatrix} 2 & -5 \\ \alpha & -2 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & \alpha \\ 1 & -2 \end{bmatrix}$