Discussion: Maple and Direction Fields

The purpose of this document is to look at how Maple defines a differential equation and plots its direction field.

We will define a differential equation, then we will assign that DE to a variable in Maple that will make it easier to do things with it later.

First, we will define the derivative in terms of the `diff` command:

Math notation:	Maple notation:
$y'(t)$	`diff(y(t),t)`
$y'(x)$	`diff(y(x),x)`
$y''(t)$	`diff(y(t),t$2)`
$y'''(t)$	`diff(y(t),t$3)`

And so on. Further, if y appears in the differential equation, be sure to write it in Maple as either $y(t)$ or $y(x)$ so that Maple knows this is a function (and not a variable).

EXAMPLE 1: (See page 41 of Boyce and Diprima) Plot the direction field for

$$y' = \frac{x^2}{1-y^2}$$

First, define the differential equation and assign it to a variable (in this case, `diffeqn`). In the first example, we treat $y(x)$ as an expression in x:

```
diffeqn:=diff(y(x),x)=x^2/(1-(y(x))^2);
```

And now we can plot the direction field using the `DEplot` command. In order to use this command, we first have to tell Maple to load in the package of commands called `DEtools`. Here are the two commands:

```
with(DEtools):
DEplot(diffeqn,y(x),x=-4..4,y=-4..4);
```

You'll notice that we needed to tell Maple (1) the differential equation, (2) What the independent and dependent variables are (the statement $y(x)$ does this), and (3) the window size that we wanted to plot.

We'll now take a look at some optional commands. Try redoing the last line, and make a note at how the plot changes:

```
DEplot(diffeqn,y(x),x=-4..4,y=-4..4, title="First Plot",arrows=LARGE);
```

```
DEplot(diffeqn,y(x),x=-4..4,y=-4..4, dirgrid=[30,30]);
```

We can use the same command to plot some sample solution curves along with the direction field. To do this, we will have to include some initial values.

EXAMPLE 2: Plot the direction field for the given differential equation, together with the three sample solutions through $(1, 2), (1, 0)$ and $(1, 1)$, if

$$ty' + 2y = 4t^2$$

First, define the differential equation, then plot:

```
diffeqn:=t*diff(y(t),t)+2*y(t)=4*t^2;
DEplot(diffeqn,y(t),t=0.01..2,[[y(1)=2],[y(1)=0],[y(1)=1]],y=-2..4);
```

Output from Maple