Practice with Integrals

Work out the following antiderivatives. There is a "hint sheet" attached. For the solutions, see the Maple file online.

1. $\displaystyle\int \frac{2x^2 - x + 4}{x^3 + 4x}\,dx$
2. $\displaystyle\int e^{2\theta}\sin(3\theta)\,d\theta$
3. $\displaystyle\int \frac{1}{y(2-y)}\,dy$
4. $\displaystyle\int t^2\cos(3t)\,dt$
5. $\displaystyle\int \frac{x-1}{x^2+1}\,dx$
6. $\displaystyle\int y\sinh(y)\,dy$
7. $\displaystyle\int \frac{dx}{x^4 - x^2}$
8. $\displaystyle\int \frac{t^2}{t+4}\,dt$
9. $\displaystyle\int \frac{x-1}{x+4}\,dx$
10. $\displaystyle\int \tan^{-1}(x)\,dx$

Hint Sheet: Integration Practice

1. $\displaystyle\int \frac{2x^2 - x + 4}{x^3 + 4x}\,dx$

   Use Partial Fraction Decomposition,

$$\frac{2x^2 - x + 4}{x^3 + 4x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

   Once you've found A, B, C, break the integral up:

$$A\int \frac{1}{x}\,dx + B\int \frac{x}{x^2+4}\,dx + C\int \frac{1}{x^2+4}\,dx$$

   Use u, du substitution for the second integral, and do some algebra on the last integral so you can use the inverse tangent as the antiderivative.

2. $\displaystyle\int \mathrm{e}^{2\theta}\sin(3\theta)\,d\theta$

   Integration by parts twice so that:

$$\int \mathrm{e}^{2\theta}\sin(3\theta)\,d\theta = \mathrm{e}^{2\theta}\,(...) - \frac{4}{9}\int \mathrm{e}^{2\theta}\sin(3\theta)\,d\theta$$

   Now solve for $\int \mathrm{e}^{2\theta}\sin(3\theta)\,d\theta$.

3. $\displaystyle\int \frac{1}{y(2-y)}\,dy$

   Use partial fractions:

$$\frac{1}{y(2-y)} = \frac{A}{y} + \frac{B}{2-y}$$

   then integrate. You might write it as $A/y - B/(y-2)$ to make it easier to integrate later.

4. $\displaystyle\int t^2\cos(3t)\,dt$

   Integration by parts; put t^2 in the middle column of the table so that the third derivative is zero.

5. $\displaystyle\int \frac{x-1}{x^2+1}\,dx$

   We can write this as:

$$\int \frac{x}{x^2+1}\,dx - \int \frac{1}{x^2+1}\,dx$$

   Use u, du substitution for the first integral, the second integral is already in a nice form.

6. $\int y \sinh(y)\, dy$

Use integration by parts, where the antiderivative of $\sinh(y)$ is $\cosh(y)$, and the antiderivative of $\cosh(y)$ is $\sinh(y)$ (so put y in the middle column).

7. $\int \frac{dx}{x^4 - x^2}$

Factor the denominator, and use integration by parts. The x^2 term can be thought of as a doubled up linear factor, so we would use:

$$\frac{A}{x} + \frac{B}{x^2}$$

or as a single quadratic factor, in which case we would use:

$$\frac{Ax + B}{x^2}$$

(These are equivalent). In any event,

$$\frac{1}{x^2(x^2-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x+1}$$

8. $\int \frac{t^2}{t+4}\, dt$

The degree of the numerator is greater than the degree of the denominator. Use long division to get:

$$\frac{t^2}{t+4} = t^2 - 4 + \frac{16}{t+4}$$

9. $\int \frac{x-1}{x+4}\, dx$

The degree of the numerator is larger than or equal to the degree of the denominator, so perform long division first:

$$\frac{x-1}{x+4} = 1 - \frac{5}{x+4}$$

10. $\int \tan^{-1}(x)\, dx$

Use integration by parts, with $u = \tan^{-1}(x)$ and $dv = 1\, dx$.