# Homework for 7.1

After Tuesday's lecture, we should be able to:

- Convert a second order linear homogeneous DE to a system of first order DEs.

  EXAMPLE: $y'' + 2y' + y = 0$

  SOLUTION: Let $x_1 = y$ and $x_2 = y'$. Then $x_1' = y' = x_2$ and $x_2' = y'' = -y - 2y' = -x_1 - 2x_2$. Therefore, the system is

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -x_1 - 2x_2 \end{aligned}$$

- Convert a system of first order DEs to a second order linear homogeneous DE.

  EXAMPLE: $\begin{aligned} x_1' &= -x_1 + x_2 \\ x_2' &= 3x_1 - x_2 \end{aligned}$

  SOLUTION: Solve the first equation for $x_2$ in terms of $x_1$: $x_2 = x_1' + x_1$. Use this expression in the second equation to get an equation completely in terms of $x_1$:

$$x_2' = 3x_1 - x_2 \quad \Rightarrow \quad (x_1' + x_1)' = 3x_1 - (x_1' + x_1) \quad \Rightarrow \quad x_1'' + x_1' = 3x_1 - x_1' - x_1$$

  Giving us:

$$x_1'' + 2x_1' - 2x_1 = 0$$

- Solve a system of first order DEs ($x'(t) = ...$ and $y'(t) = ...$) by first forming $dy/dx$.

  EXAMPLE: $\begin{aligned} x' &= 2y \\ y' &= 3x \end{aligned}$

  SOLUTION:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3x}{2y} \quad \Rightarrow \quad 2y\,dy = 3x\,dx \quad \Rightarrow \quad y^2 = \frac{3}{2}x^2 + C$$

## More Practice Problems

1. Convert to a system of first order DEs:
   (a) $y'' + 9y = 0$
   (b) $y'' + 6y' + 9y = 0$
   (c) $y'' + y' - 2y = 0$
2. For the previous problems, solve the systems by using methods from Chapter 3 on the second order equation.
3. Convert the following systems to second order equations:

(a) $\begin{aligned} x_1' &= 2x_1 + x_2 \\ x_2' &= x_1 + x_2 \end{aligned}$

(b) $\begin{aligned} x_1' &= x_2 \\ x_2' &= 3x_1 + x_2 \end{aligned}$

(c) $\begin{aligned} x_1' &= x_1 \\ x_2' &= x_1 + x_2 \end{aligned}$

4. Solve the system by first writing the DE as $dy/dx$:

(a) $\begin{aligned} x' &= x \\ y' &= x + y \end{aligned}$

(How else might you solve this system more directly?)

(b) $\begin{aligned} x' &= x + y \\ y' &= x - y \end{aligned}$

(For this problem, is the DE homogeneous (the Chapter 2 version)? If so, just write how you would proceed to solve it).