Example HW from 3.6

Exercises 13-20 are paired with exercises 21-28. The first set translates the model second order equation into a system of first order, then uses the ideas from Chapter 3 to solve it. The second set, 21-28, is to look at the equation as a second order DE, and solve it directly. Here is an example of Exercise 14 and 22:

14 Model equation is $y'' + 6y' + 8y = 0$. For the system, let $x_1 = y$ and $x_2 = y'$ so that the system is given by:

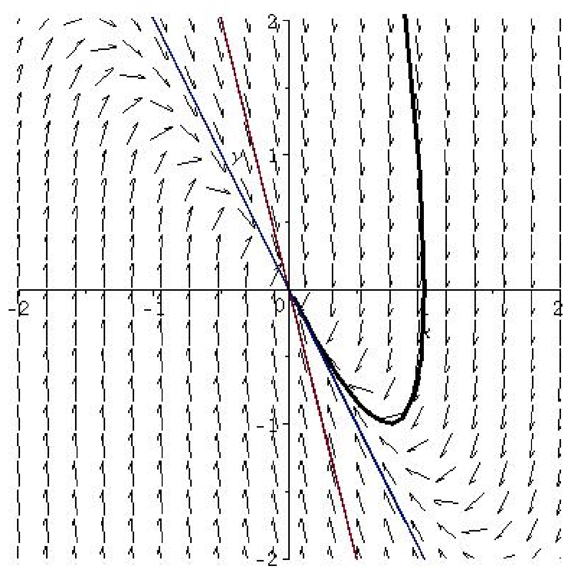
$$\begin{array}{ll} x_1' = & x_2 \\ x_2' = -8x_1 & -6x_2 \end{array} \quad \Rightarrow \quad \mathbf{x}' = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} \mathbf{x}$$

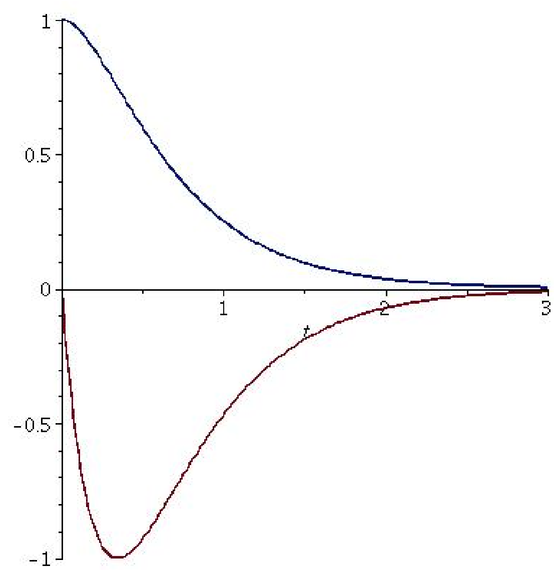
For part (b), the eigenvalues are the solutions to $\lambda^2 + 6\lambda + 8 = 0$, or $\lambda = -2, -4$. For each λ, find the eigenvector:

$$\lambda_1 = -2 \quad \Rightarrow \quad 2v_1 + v_2 = 0 \quad \Rightarrow \quad \begin{array}{ll} v_1 & = v_1 \\ v_2 & = -2v_1 \end{array} \quad \Rightarrow \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\lambda_2 = -4 \quad \Rightarrow \quad 4v_1 + v_2 = 0 \quad \Rightarrow \quad \begin{array}{ll} v_1 & = v_1 \\ v_2 & = -4v_1 \end{array} \quad \Rightarrow \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

For part (c), the system is overdamped. For the graphs, sketch the eigenvectors first. In this case, there is a sink- Notice that e^{-4t} goes to zero very quickly, so that the solution will approach the origin tangent to the other eigenvector, $[1, -2]^T$. The second graph is going to depend on the first, so try to get a good sketch for the first part anyway.





22 The general solution is given by:

$$y(t) = C_1 e^{-2t} + C_2 e^{-4t}$$

The particular solution is given by:

$$y(t) = 2e^{-2t} - e^{-4t}$$

The graphs are in the previous exercise.