Review for Exam 3

Section 2.8, while assigned, will not be on the exam. The exam will cover sections 3.6, 4.1-4.4, and 5.1-5.2. Notice that 3.6 and 4.1-4.4 are all about a detailed understanding of the second order linear differential equation:

$$ay'' + by' + cy = F(t)$$

While 5.1 and 5.2 got us back into systems of equations, and showed us how to use linear analysis in nonlinear differential equations. As usual, the exam will be 50 minutes in length, and you will not be allowed calculators or notes.

Topic List

- 3.6: Second order (homogeneous) DEs, $ay'' + by' + cy = 0$.
  - Vocab: Natural frequency, simple harmonic motion, underdamped, overdamped, critically damped.
  - Technique: How to solve using the characteristic equation directly (without writing it as a system, three cases).

    Note: We should also be able to write the DE as a *system* and analyze through eigenvalues and eigenvectors, and via the Poincaré Diagram. This was explored in exercises 13-20, while the new techniques were used in 21-28.
- 4.1: $ay'' + by' + cy = F(t)$ for certain $F(t)$.
  - Vocab: Forced response, steady-state response, natural response (or free response).
  - Theory: We can break the solution up into pieces - The homogeneous and the particular. If $F(t)$ contains a sum, we can further break up the particular part of the solution. This is the extended linearity principle.
  - Technique: Method of Undetermined Coefficients. Note that periodic forcing was dealt with in the next two sections, where we "complexified" the problem.
  - Note that, if $a, b, c > 0$ (which they typically are in the oscillator model), then the homogeneous part of the solution will always go to zero as $t \to \infty$, which leaves the forced response $y_p(t)$.
- 4.2: $y'' + py' + qy = \cos(\omega t)$ (or we could have $\sin(\omega t)$ for forcing).
  - Technique: Be able to write $A\cos(\omega t) + B\sin(\omega t) = R\cos(\omega t - \delta)$.
  - Technique: This section introduced the idea of "complexification". Besides using it in Method of Undetermined Coefficients, the method works in integration problems and in trig identities. See if you can find the trig identities for $\cos(x - y)$ and $\sin(x - y)$ using complexification.

- Graphical analysis: Be able to analyze graphs as solutions to certains DEs (as in #17).

- 4.3: No damping, periodic forcing leads to beating and resonance.
  - Given $y'' + b^2 y = \cos(\omega t)$, we have beating when $\omega \approx b$. The period of the beat is $2\pi/|\omega - b|$ and the amplitude is $2/|b^2 - \omega^2|$.
    Why is this important? It tells us what happens as $\omega \to b$- The period of the beats gets longer and longer, and the amplitude gets larger and larger, until we get **resonance**.
  - Be able to give the analytic solution to $y'' + b^2 y = \cos(\omega t)$, where $b \neq \omega$.
  - Be able to solve the resonance equation: $y''+b^2 y = \cos(bt)$, either using l'Hospital's rule (like we did in class), or by Method of Undetermined Coefficients.
  - Be able to do a bit of graphical analysis, as we did in Exercise 21.

- 4.4: Damping and periodic forcing: $y'' + py' + qy = \cos(\omega t)$
  - Be able to solve the system analytically. Be able to just give the amplitude and period of the forced response without having to fully solve the DE. For relevant problems, see the homework handout that was assigned (the solutions are posted on the class website).
  - Understand that, even though damping is now present, it is still possible to have resonance- which in this case is having a periodic forcing function that yields a response with amplitude much higher than might otherwise be expected (especially as the amount of damping gets very small).
  - Be able to determine (using derivatives) the value of ω that will maximize the response.

- 5.1: Equilibrium point analysis
  - Vocab: Linearize, Jacobian, separatrix (stable and unstable)
  - Be able to linearize a given function (of one or two variables) about a given point.
  - Be able to linearize a system of differential equations about a given point. This uses the Jacobian matrix (of first partial derivatives).
  - Understand when linearization fails (two things to watch for- Centers and zero eigenvalues).
  - Main idea: Linearize about all equilibria, and identify what kind of equilibrium is at the center (using Poincaré).

- Qualitative analysis (using nullclines)

  All about the use of nullclines to help analyze a system of differential equations.

I won't ask you to find eigenvalues/eigenvectors of anything except a 2×2 matrix.