## Visualizing Solutions

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**Definition:** A direction field is a plot in the $(t, y)$ plane that give the local tangent lines to the solution to a first order ODE.

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The points that correspond to a slope of $-2$:

$$-2 = t - y^2 \quad \Rightarrow \quad t = y^2 - 2$$

and so on...

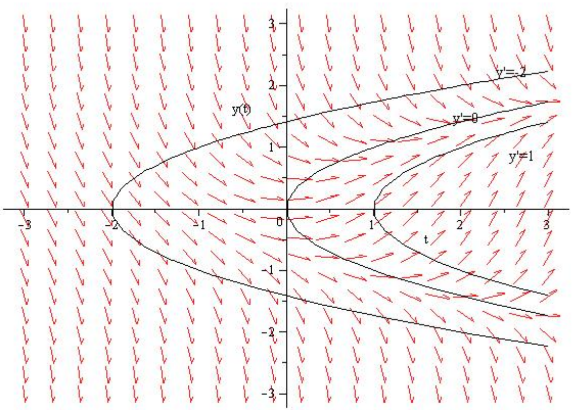


Figure: Direction Field with Isoclines: $y' = -2, y' = 0, y' = 1$

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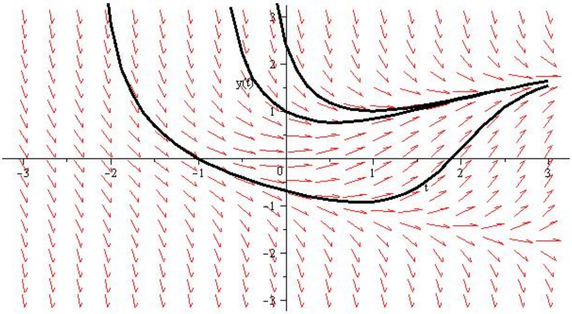
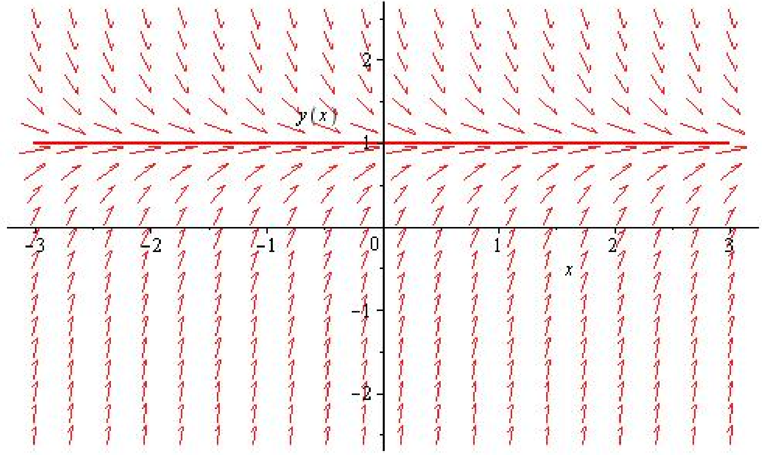


Give an ODE of the form $y' = ay + b$ whose direction field looks like:



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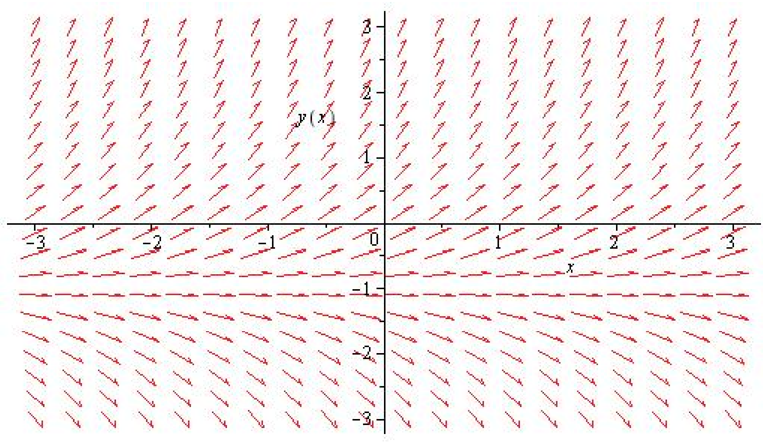
SOLUTION: $y' = 1 - y$

Same question as before:



# Choose a DE

1. $y' = 3 - y$
2. $y' = y(y+3)$
3. $y' = y(3-y)$
4. $y' = 2y - 1$