Math 244 Sample Final A

Show all your work! A table of Laplace transforms is provided.

1. Find values of k for which the IVP: $ty' - 4y = 0$, $y(0) = k$ has (i) No solution, (ii) An infinite number of solutions. Does this violate the Existence and Uniqueness Theorem (explain)?

2. Suppose you have a tank of brine containing 300 gallons of water with a concentration of 1/6 pounds of salt per gallon. There is brine pouring into the tank at a rate of 3 gallons per minute, and it contains 2 pounds of salt per gallon. The well-mixed solution leaves at 2 gallons per minute. (i) Write the initial value problem for the amount of salt in the tank at time t, and (ii) solve it.

3. For the following, find the power series expansion for the general solution up to and including the t^4 term:
$$y' - 2y = \sin(t)$$

4. Using the method of undetermined coefficients, give the form of the particular solution (do not solve) to:
$$y'' - 6y' + 9y = 6t^2 - 12te^{3t}$$

5. Classify the origin using the Poincaré Diagram and solve using eigenvectors/eigenvalues, then provide a sketch of the phase portrait:
$$\mathbf{Y}'(t) = \begin{bmatrix} -1 & 2 \\ -2 & -1 \end{bmatrix} \mathbf{Y}$$

6. Solve:
   (a) $y' = -\frac{y}{1+t} + t^2$
   (b) $y' = y(3 - 2y)$
   (c) $t\dfrac{dy}{dt} - (1+t)y = ty^2$ First, use the substitution: $u = y^{-1}$ to get a DE in u.

7. Solve for the Laplace Transform, $Y(s)$, of the solution $y(t)$ (do not invert the transform):
$$y'' + 6y' + 5y = t - t^2 u_2(t), \quad y(0) = 1, y'(0) = 0$$

8. Compute $\mathcal{L}^{-1}\left(\dfrac{s}{s^2 - 10s + 29}\right)$

9. Write the solution to the following DE in terms of $g(t)$: $y'' + 4y = g(t)$, $y(0) = 3, y'(0) = -1$.

10. Given the system of equations below, describe (using the Poincaré Diagram) how the classification of the origin changes with α.
$$\mathbf{Y}' = \begin{bmatrix} \alpha & 1 \\ -2 & 0 \end{bmatrix} \mathbf{Y}$$

11. Suppose that our mass-spring system is given by $y'' + 3y' + y = \cos(\omega t)$.
    (a) Is there any value of ω that would give us resonance? Beating?
    (b) Find the value of ω that gives the maximum amplitude for the particular solution.

12. The graph below is $y' = f(y)$.

   (a) Locate and classify all equilibria.

   (b) Provide a sketch of the direction field.

   (c) Give one interval on which $y(t)$ is concave up.

   (d) True or False? The solution $y(t)$ may be periodic.