Math 244: More Practice with Modeling

1. A mass weighing 4 lb stretches a spring 2 in. Suppose that the mass is displaced an additional 6 in the positive direction and released. The mass is in a medium that exerts a viscous resistance of 6 lb when the mass has a velocity of 3 ft/s. Formulate the IVP that governs the motion of the mass. (Hint: All units should be consistent. When working with US units, use pounds, feet and seconds.)

2. Suppose we consider a mass-spring system with no damping (the damping constant is then 0), so that the differential equation expressing the motion of the mass can be modeled as

$$mu'' + ku = 0$$

Find value(s) of β so that $A\cos(\beta t)$ and $B\sin(\beta t)$ are each solutions to the homogeneous equation (for arbitrary values of A, B).

3. A mass weighing 2 lb stretches a spring 6 in. If the mass is pulled down an additional 3 in and then released, and if there is no damping, determine the IVP that governs the motion of the mass. (You might re-read the hint for (1))

4. A mass of 20 grams stretches a spring 5 cm. Suppose tha the mass is attached to a viscous damper with a constant damping constant of 400 dyn-s/cm (note: a dyne is a unit of force using centimeters-grams- seconds for units). If the mass is pulled down an additional 2 cm then released, find the IVP that governs the motion of the mass.

5. For exercises 1, 3 and 4 above, convert the mass-spring second order DE into a system of first order equations, then analyze the solutions using the HPGSystemSolver software (in DETools).

6. A mass weighing 8 lbs stretches a spring $\frac{3}{2}$ in. The mass is attached to a damper with coefficient γ. Using the ansatz to get the characteristic equation for the second order linear, homogeneous, DE, find the value of γ at which we get one single real solution (to the characteristic equation).