Project 1: The Learning Model

In this assignment, we want to use all three methods for exploring the solutions to a differential equation: Graphical, Numerical and Analytic.

The general model we'll be considering is the learning model (discussed on the bottom of page 17 and top of page 18):

$$\frac{dL}{dt} = k(1 - L)$$

Answer the following questions. Be sure to give a full answer (in complete sentences); do NOT just give the numerical result. This was designed so that you can use the `HPGSolver` software and the online Euler's Method Calculator (linked from the website), or you can use any other software to help (like Mathematica or Maple). The differential equation is given as:

$$\frac{dL}{dt} = k(1 - L)$$

Turn in your handwritten write-up that includes nicely laid out graphics! **Due: At the beginning of class, Monday, Feb 5th.** This project will be counted with the quizzes.

Graphical Analysis

Plot the DE in the (L, L') plane and give an analysis. Have a couple of different graphs that depend on k. Each should correspond to a graph in the (t, L) plane. In your analysis, discuss the equilibrium solution, and at what point learning takes place most rapidly. Finally, answer the question: If one student knows $1/2$ of the list at time $t = 0$, and a second student starts knowing nothing, does our model ever allow the second student to "overtake" the first? Why?

Numerical Analysis

Use Euler's Method to give some sample solutions, changing the value of k and Δt. In particular, take note of $L(2)$ in each case.

Analytic Solution

Solve the differential equation so that $L(0) = L_0$. Using this function and the values of k from the numerical section, compare the actual solution at time 2 with the approximated solution at time 2. What do you notice?

HINT: You might make a table like this:

	$k = 0.5$	$k = 1$	$k = 2$
$\Delta t = 0.5$			
$\Delta t = 0.2$			